A GENERALIZED NET FOR MACHINE LEARNING OF THE PROCESS OF MATHEMATICAL PROBLEMS SOLVING
On an Example with a Smarandache Problem

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The authors of the present paper prepared a series of research related to the ways of representation by Generalized nets (GNs, see [1] and the Appendix) the process of machine learning of different objects, e.g., neural networks, genetic algorithms, GNs, expert systems, systems (abstract, statical, dynamical, stohastical and others), etc. Working on their research [2], where they gave a counterexample of the 62-nd Smarandache's problem (see [3]), they saw that the process of the machine learning of the process of the mathematical problems solving also can be described by a GN and by this reason the result form [2] was used as an example of the present research. After this, they saw that the process of solving of a lot of the Smarandache's problems can be represented by GNs in a similar way and this will be an object of next their research.

The GN (see [1] and the Appendix), which is described below have three types of tokens $\alpha-$, $\beta-$ and $\gamma-$ tokens. They interprete respectively the object which will be studied, its known property (properties) and the hypothesis, related to it, which must be checked. The tokens' initial characteristics correspond to these interpretations. The tokens enter the GN, respectively, through places

- $l_1$ with the initial characteristic "description of the object" (if we use the example from [2], this characteristic will be, e.g., "sequence of natural numbers")
- $l_2$ with the initial characteristic "property (properties) of the object, described as an initial characteristic of $\alpha-$token corresponding to the present $\beta-$token" (in the case of the example mentioned above, it will be the following property "there are no three elements of the sequence, which are members of an arithmetic progression") and
- $l_3$ with the initial characteristic "description of an hypothesis about the object" (for the discussed example this characteristic will be, e.g., "the sum of the reciprocal values of the members of the sequence are smaller than 2")

We shall would like for the places' priorities to satisfy the following inequalities:

$$\pi_L(l_1) > \pi_L(l_2) > \pi_L(l_3),$$
$$\pi_L(l_4) > \pi_L(l_5) > \pi_L(l_6),$$

The GN transitions (see [1] and the Appendix) have the following forms:

$$Z_1 = \{\{l_1, l_2, l_3, l_{21}, l_{22}, l_{25}\}, \{l_4, l_5, l_6\}, r_1, M_1, \Box_1\},$$

where

<table>
<thead>
<tr>
<th></th>
<th>$l_4$</th>
<th>$l_5$</th>
<th>$l_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$</td>
<td>true</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>$l_2$</td>
<td>false</td>
<td>true</td>
<td>false</td>
</tr>
<tr>
<td>$l_{21}$</td>
<td>true</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>$l_{22}$</td>
<td>false</td>
<td>true</td>
<td>false</td>
</tr>
<tr>
<td>$l_{25}$</td>
<td>false</td>
<td>false</td>
<td>true</td>
</tr>
</tbody>
</table>

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\[
\begin{array}{c|ccc}
 & l_4 & l_5 & l_6 \\
 l_1 & 1 & 0 & 0 \\
 l_2 & 0 & 1 & 0 \\
 M_1 = & l_3 & 0 & 0 & 1 \\
 & l_{21} & 1 & 0 & 0 \\
 & l_{23} & 0 & 1 & 0 \\
 & l_{25} & 0 & 0 & 1 \\
\end{array}
\]

\[k v_1 = \land (v(l_1, l_{21}), v(l_2, l_{23}), v(l_3, l_{25})).\]

The \(\alpha-\)token obtains the characteristic "the initial status of the object, having in mind the current \(\gamma-\)characteristic" in place \(l_4\), the \(\beta-\)token obtains the characteristic "a next state of the object, having in mind the current \(\alpha-\) and \(\gamma-\)characteristics" in place \(l_5\), and the \(\gamma-\)token obtains the characteristics "restrictions over the object, having in mind its property (properties) from the initial \(\beta-\)characteristic in place \(l_6\). For the discussed example with the 62-nd Smarandache's problem, the last three characteristics have the following forms, respectively: "1, 2" (initial values of the sequence); "3" (next value of the sequence); e.g., "the members to be minimal possible".

\[Z_2 = \langle \{l_5, l_7\}, \{l_7, l_8\}, r_2, M_2, v(l_5, l_7) \rangle,\]

where

\[
\begin{array}{c|cc}
 & l_7 & l_8 \\
 r_2 = & l_5 & r_{5,7} \quad r_{5,8} \\
 & l_7 & r_{7,7} \quad r_{7,8} \\
\end{array}
\]

where

\(r_{5,7} = r_{7,7} = "\text{the new state of the object does not satisfy the property of the object determined by the initial } \beta-\text{characteristic}"\)

\(r_{5,8} = r_{7,8} = \neg r_{5,7}.\) and

\[
\begin{array}{c|cc}
 & l_7 & l_8 \\
 M_3 = & l_5 & 1 & 1 \\
 & l_7 & 1 & 1 \\
\end{array}
\]

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The \( \beta \)-token obtains the characteristic "a next state of the object, having in mind the current \( \alpha \)- and \( \gamma \)-characteristics" in place \( l_7 \) and it does not obtain any characteristic in place \( l_4 \). In the case of our example, on the first time, when the \( \beta \)-token enters place \( l_7 \) will obtain the characteristic "4".

\[
Z_3 = \{\{l_4, l_6, l_8\}, \{l_9, l_{10}, l_{11}, l_{12}, l_{13}, l_{14}\}, r_3, M_3, \square_3, \}
\]

where

\[
\begin{array}{c|cccccc}
 & l_9 & l_{10} & l_{11} & l_{12} & l_{13} & l_{14} \\
\hline
l_4 & r_{4,9} & r_{4,10} & false & false & false & false \\
l_6 & false & false & r_{6,11} & r_{6,12} & false & false' \\
l_8 & false & false & false & false & r_{8,13} & r_{8,14} \\
\end{array}
\]

where

\[
r_{4,9} = r_{6,11} = r_{8,13} = "the new state is not a final one", \\
r_{4,10} = r_{6,12} = r_{8,14} = \neg r_{4,9},
\]

\[
M_3 = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 \\
\end{bmatrix},
\]

\[
\square_3 = \land(l_4, l_6, l_8).
\]

The \( \alpha \)-token does not obtain any characteristic in place \( l_9 \), and it obtains the characteristic "the list of all states of the object" in place \( l_{10} \); the \( \beta \)-token does not obtain any characteristic in places \( l_{11} \) and \( l_{12} \); the \( \gamma \)-token does not obtain any characteristic in place \( l_{13} \), and it obtains the characteristic

\[
\begin{cases}
& "the hypothesis is valid by the present step", & \text{if the last state of the object satisfies the hypothesis} \\
& "the hypothesis is not valid by the present step", & \text{if the last state of the object does not satisfy the hypothesis}
\end{cases}
\]

in place \( l_{14} \). For the discussed example with the 62-nd Smarandache's problem, the tokens do not obtain any characteristics.

\[
Z_4 = \{\{l_9, l_{11}, l_{13}\}, \{l_{15}, l_{16}, l_{17}, l_{18}, l_{19}, l_{20}\}, r_4, M_4, \square_4, \}
\]

where

\[
\begin{array}{c|cccc}
 & l_{15} & l_{16} & l_{17} & l_{18} & l_{19} & l_{20} \\
\hline
l_9 & r_{9,15} & r_{9,16} & false & false & false & false \\
l_{11} & false & false & r_{11,17} & r_{11,18} & false & false' \\
l_{13} & false & false & false & false & r_{13,19} & r_{13,20} \\
\end{array}
\]

where

\[
r_{9,15} = r_{11,17} = r_{13,19} = "the hypothesis is valid", \\
r_{9,16} = r_{11,18} = r_{13,20} = \neg r_{9,15},
\]

\[
M_4 = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 \\
\end{bmatrix},
\]

\[
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\]
The $\alpha$-token does not obtain any characteristic in place $l_{15}$, and it obtains the characteristic “the final state, which violate the hypothesis” in place $l_{16}$; the $\beta$-token does not obtain any characteristic in places $l_{17}$ and $l_{18}$; the $\gamma$-token does not obtain any characteristic in place $l_{19}$, and it obtains the characteristic “the hypothesis is not valid” in place $l_{20}$. For the discussed example with the 62-nd Smarandache’s problem, the tokens do not obtain any characteristics.

\begin{equation}
Z_5 = \{l_{15}, l_{17}, l_{19}\}, \{l_{21}, l_{22}, l_{23}, l_{24}, l_{25}, l_{26}\}, r_5, M_5, \Box_5,
\end{equation}

where

\begin{equation}
r_5 = \begin{pmatrix}
l_{21} & r_{15,21} & r_{15,22} & false & false & false & false \\
l_{17} & false & false & r_{17,23} & r_{17,24} & false & false \\
l_{19} & false & false & false & r_{19,25} & r_{19,26}
\end{pmatrix}
\end{equation}

where

\begin{equation}
r_{15,21} = r_{17,23} = r_{19,25} = "\text{there is a possibility for a change of the restrictions over the object, which evolve from the hypothesis}"; \end{equation}

\begin{equation}
r_{15,22} = r_{17,24} = r_{19,26} = r_{15,21},
\end{equation}

\begin{equation}
M_3 = \begin{pmatrix}
l_{21} & l_{22} & l_{23} & l_{24} & l_{25} & l_{26} \\
l_{15} & 1 & 1 & 0 & 0 & 0 \\
l_{17} & 0 & 1 & 1 & 0 & 0 \\
l_{19} & 0 & 0 & 0 & 1 & 1
\end{pmatrix}
\end{equation}

$\Box_3 = \land(l_{15}, l_{17}, l_{19})$.

\begin{equation}
\Box_3 = \land(l_{15}, l_{17}, l_{19}).
\end{equation}

The $\alpha$-token obtains as its current characteristic its initial characteristic in place $l_{21}$, and it does not obtain any characteristic in place $l_{22}$; the $\beta$-token obtains as its current characteristic its initial characteristic in place $l_{23}$ and it does not obtain any characteristic in place $l_{24}$; the $\gamma$-token obtains the characteristic “new restrictions over the object” in place $l_{25}$, and it does not obtain any characteristic in place $l_{26}$. For the discussed example with the 62-nd Smarandache’s problem, the tokens do not obtain any characteristics in places $l_{21}, l_{23}, l_{25}$ and $l_{26}$, and the $\gamma$-token will obtain as a characteristic “1,3” $l_{25}$. These two numbers will be initial for the next search of a sequence, which satisfy the hypothesis. In the next step they will be changed, e.g., by numbers 2 and 3, etc.

Using this scheme, it is possible to describe the process of solving of some of the other Smarandache’s problems, too, e.g., problems ... from [3].

**APPENDIX: Short remarks on Generalized Nets (GNs)**

The concept of a Generalized Net (GN) is described in details in [1], see also

[www.daimi.aau.dk/PetriNets/bibl/aboutpnbibl.html](www.daimi.aau.dk/PetriNets/bibl/aboutpnbibl.html)

They are essential extensions of the ordinary Petri nets. The GNs are defined in a way that is principally different from the ways of defining the other types of Petri nets. When some of
the GN-components of a given model are not necessary, they can be omitted and the new
nets are called reduced GNs. Here a reduced GN without temporal components is used.

Formally, every transition (or the used form of reduced GN) is described by a five-tuple:
\[ Z = (L', L'', r, M, ∅), \]
where:

- \( L' \) and \( L'' \) are finite, non-empty sets of places (the transition's input and output
  places, respectively); for the above transition these are
\[ L' = \{l'_1, l'_2, \ldots, l'_m\} \]
and
\[ L'' = \{l''_1, l''_2, \ldots, l''_n\}; \]

- \( r \) is the transition's condition determining which tokens will pass (or transfer) from
  the transition's inputs to its outputs; it has the form of an Index Matrix (IM; see [1]):
\[
\begin{array}{c|cccc}
& l''_1 & \ldots & l''_j & \ldots & l''_n \\
\hline
l'_1 & \cdot & \cdot & r_{i,j} & \cdot & \cdot \\
\vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\
l'_i & \cdot & \cdot & (r_{i,j} \text{ - predicate}) & \cdot & \cdot \\
\vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\
l'_m & \cdot & \cdot & \cdot & \cdot & \cdot \\
\end{array}
\]
\( r_{i,j} \) is the predicate which corresponds to the \( i \)-th input and \( j \)-th output places. When its
truth value is "true", a token from the \( i \)-th input place can be transferred to the \( j \)-th output
place; otherwise, this is not possible;

- \( M \) is an IM of the capacities of transition's arcs:
\[
\begin{array}{c|cccc}
& l''_1 & \ldots & l''_j & \ldots & l''_n \\
\hline
l'_1 & \cdot & \cdot & m_{i,j} & \cdot & \cdot \\
\vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\
l'_i & \cdot & \cdot & (m_{i,j} \geq 0 \text{ - natural number}) & \cdot & \cdot \\
\vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\
l'_m & \cdot & \cdot & \cdot & \cdot & \cdot \\
\end{array}
\]
\( m_{i,j} \) is a natural number;

- \( ∅ \) is an object having a form similar to a Boolean expression. It may contain as
  variables the symbols which serve as labels for transition's input places, and is an expression

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built up from variables and the Boolean connectives \( \land \) and \( \lor \), with semantics defined as follows:

\[
\land(l_1, l_2, \ldots, l_n) \quad \text{every place } l_1, l_2, \ldots, l_n \text{ must contain at least one token,}
\]

\[
\lor(l_1, l_2, \ldots, l_n) \quad \text{there must be at least one token in all places } l_1, l_2, \ldots, l_n \text{, where } \{l_1, l_2, \ldots, l_n\} \subseteq L'.
\]

When the value of a type (calculated as a Boolean expression) is "true", the transition can become active, otherwise it cannot.

The object

\[
E = (A, \pi_L, K, X, \Phi)
\]

is called a (reduced) GN, if

(a) \( A \) is a set of transitions;

(b) \( \pi_L \) is a function giving the priorities of the places, i.e., \( \pi_L : L \rightarrow N \), where \( L = \text{pr}_1A \cup \text{pr}_2A \), and \( \text{pr}_iX \) is the \( i \)-th projection of the \( n \)-dimensional set, where \( n \in \mathbb{N}, n \geq 1 \) and \( 1 \leq k \leq n \) (obviously, \( L \) is the set of all GN-places);

(c) \( K \) is the set of the GN's tokens;

(d) \( X \) is the set of all initial characteristics the tokens can receive when they enter the net;

(e) \( \Phi \) is a characteristic function which assigns new characteristics to every token when it makes the transfer from an input to an output place of a given transition.

REFERENCES: