Mike Mudge pays a return visit to the Florentin Smarandache Function. 1

The originator of this function, Florentin Smarandache, an Eastern European mathematician, escaped from the country of his birth because the Communist authorities had prohibited the publication of his research papers and his participation in international congresses. After spending two years in a political refugee camp in Turkey, he emigrated to the United States.


PCW readers may have met this function before, in Numbers Count -112- February 1992, where a very encouraging response was generated. This article (February 1993) is complete in itself so don’t worry if you have filed the July issue! It may be thought that those readers who attempted the previous problem-set will have an unfair advantage. However, it must be realised that no Numbers Count problems are completely original so previous work within a given subject area is always a possibility and the prize is awarded using suitable subjective criteria anyway, so please have a go and submit your results, however trivial they may seem to yourself.

Definition For all non-null integers, n, the Smarandache Function, S(n), is defined to be the smallest integer such that (S(n))! (The Factorial Function with argument S(n)) is divisible by n, e.g. S(18) = 6 because 6! is divisible by 18 but 1!,...,5! are not.

Problem (i) Design and implement an algorithm to generate and store/tabulate S(n) as a function of n up to a given number M.

Hint It may be advantageous to consider the STANDARD FORM of n, viz n = e pt.pq,...,pr where e=1, and p,p,p, denote the distinct prime factors of n and a,a,...a, are their respective multiplicities.

NOTE S(n) is an even function, by which is meant S(-n) = S(n).

Problem (ii) For a specified non-null integer m, under what conditions does S(n) divide the difference n - m?

Problem (iii) Investigate the possible existence of integer solutions (x,y,z) of S(x) + S(y) = S(z) + S(z) for any n greater than or equal to 1. e.g. examine the solution (S,7,2048) when n = 3.

It can be proved that an infinity of solutions exist for any such n-value.

Problem (iv) Investigate the possibility of finding two integers n and k such that the LOGARITHM of S(n) to the BASE S(k) is an integer.

Problem (v) Recall that ‘Gamma’ defined as the limit as n tends to infinity of (1 + 1/2 + 1/3 + 1/4,...,1/n - log(n)) exists, is known as Euler’s Constant and is approximately 0.577.

Investigate the possible existence of ‘Samma’ defined as the limit as n tends to infinity of (1 + 1/S(2) + 1/S(3)....,1/S(n) - log(S(n)).

Problem (vi) Find the number of PARTITIONS of n as the sum of S(m) for 2 ≤ m ≤ n.

Review of ‘Numbers Count -118- February 1993: a revisit to The Florentin Smarandache Function’ 2

This produced a number of quite powerful responses. As a note of related interest, the latest publication of F. Smarandache is ‘A Numerical Function in Congruence Theory’, Libertas Mathematica (American Romanian Academy of Arts and Science) vol. 12, 1992, pp 181-185. Arlington, Texas.

Pal Gronas of Norway submitted theoretical results on both problems 0 & (v). However, the clear winner this month is a former regular respondent, now retired, Henry Ibsenity, Glimminge 2036, 280 60 Broby, Sweden. Henry used a dsk-computer with 486/33MHz processor in Borland’s Turbo Basic. S(n) up to 10^9 took 2hr 50min. He completed a great deal of work on all problems except (vi): details of numerical results and conclusions available from Henry or myself to interested readers. What about problem (vi)?

1 Republished from <Personal Computer World>, No.118, 403, February 1993 (with the author permission), because some of the following research papers are referring to these open problems.

2 Republished from <Personal Computer World>, No.124, 495, August 1993 (with the author permission).