The main objective of this note is to introduce the notion of the S-multiplicative function and to give some simple properties concerning it. The name of S-multiplicative is short for Smarandache-multiplicative and reflects the main equation of the Smarandache function.

Definition 1. A function $f: \mathbb{N}^* \rightarrow \mathbb{N}^*$ is called S-multiplicative if:

1. $f(a \cdot b) = \max \{ f(a), f(b) \}$

The following functions are obviously S-multiplicative:

1. The constant function $f: \mathbb{N}^* \rightarrow \mathbb{N}^*$, $f(n) = 1$.
2. The Erdos function $f: \mathbb{N}^* \rightarrow \mathbb{N}$, $f(n) = \max \{ \pi(p) : p \text{ is prime and } n \div p \}$. [1].
3. The Smarandache function $S: \mathbb{N}^* \rightarrow \mathbb{N}$, $S(n) = \max \{ p'! : n \}$. [3].

Certainly, many properties of multiplicative functions[2] can be translated for S-multiplicative functions. The main important property of this function is presented in the following.

Definition 2. If $f: \mathbb{N}^* \rightarrow \mathbb{N}$ is a function, then $\bar{f}: \mathbb{N}^* \rightarrow \mathbb{N}$ is defined by

$$\bar{f}(n) = \min \{ f(d) : n \div d \}.$$

Theorem 1. If $f$ is S-multiplicative function, then $\bar{f}$ is S-multiplicative.

Proof. This proof is made using the following simple remark:

$$\max \{ \min \{ f(d_1), f(d_2) \} : d \div n \} = \min \{ \max \{ f(d_1), f(d_2) \} : d \div n \}.$$

Let $a, b$ be two natural numbers, such that $(a, b) = 1$. Therefore, we have

$$(3) \; \bar{f}(a \cdot b) = \min \{ f(d) : d \div a, d \div b \} = \min \{ f(d_1), f(d_2) \} = \min \{ \max \{ f(d_1), f(d_2) \} : d \div n \}.$$

Applying the distributing property of the max and min functions, equation (3) is transformed as follows:

$$\bar{f}(a \cdot b) = \max \{ \min \{ f(d_1), f(d_2) \} : d \div n \} = \max \{ \bar{f}(a), \bar{f}(b) \}.$$

Therefore, the function $\bar{f}$ is S-multiplicative.

We believe that many other properties can be deduced for S-multiplicative functions. Therefore, it will be in our attention to further investigate these functions.

References
