ALGORITHM FOR LISTING OF SMARANDACHE FACTOR PARTITIONS

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ABSTRACT: In [1] we define SMARANDACHE FACTOR PARTITION FUNCTION (SFP), as follows:

Let \( \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_r \) be a set of \( r \) natural numbers and \( p_1, p_2, p_3, \ldots, p_r \) be arbitrarily chosen distinct primes then \( F(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_r) \) called the Smarandache Factor Partition of \( (\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_r) \) is defined as the number of ways in which the number \( N = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \ldots p_r^{\alpha_r} \) could be expressed as the product of its' divisors. For simplicity, we denote \( F(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_r) = F'(N) \) ,where

\[
N = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} \ldots p_r^{\alpha_r} p_n^{\alpha_n}
\]

and \( p_r \) is the \( r \text{th} \) prime. \( p_1 = 2, p_2 = 3 \) etc.

In this note an algorithm to list out all the SFPs of a number without missing any is developed.

DISCUSSION:

DEFINITION: \( F'_x(y) \) is defined as the number of those SFPs of \( y \) which involve terms not greater than \( x \).

If \( F_1 \) be a factor partition of \( y \):

\[
F_1 \rightarrow x_1 X x_2 X x_3 X \ldots X x_r , \text{ then } F_1 \text{ is included in } F'_x(y) \text{ iff }
\]

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\[ x_i \leq x \quad \text{for} \quad 1 \leq i \leq r \]

Clearly \( F'_x(y) \leq F'(y) \), The equality holds good \( \text{iff} \ x \geq y \).

Example: \( F'_8(24) = 5 \). Out of 7 only the last 5 are included in \( F'_8(24) \).

(1) 24
(2) 12 X 2
(3) 8 X 3
(4) 6 X 4
(5) 6 X 2 X 2
(6) 4 X 3 X 2
(7) 3 X 2 X 2 X 2.

**ALGORITHM:** Let \( d_1, d_2, d_3, \ldots, d_r \) be the divisors of \( N \) in descending order. For listing the factor partitions following are the steps:

(A) (1) Start with \( d_1 = N \). 
(2) Write all the factor partitions involving \( d_2 \) and so on.
(B) While listing care should be taken that the terms from left to right should be written in descending order.

** At \( d_k \geq N^{1/2} \geq d_{k+1} \), and onwards, step (B) will ensure that there is no repetition.

**Example:** \( N = 36 \), Divisors are 36, 18, 12, 9, 6, 4, 3, 2, 1.

\[
\begin{align*}
36 & \rightarrow 36 \\
18 & \rightarrow 18 \times 2 \\
12 & \rightarrow 12 \times 3 \\
9 & \rightarrow 9 \times 4 \\
& \quad 9 \times 2 \times 2 \\
6 & \rightarrow 6 \times 6 \\
6 & \rightarrow 6 \times 3 \times 2 \\
\hline d_k = N^{1/2}
\end{align*}
\]

4 \( \rightarrow 4 \times 3 \times 3 \)
FORMULA FOR $F'(N)$

$$F'(N) = \sum_{d_r/N} F'_{d_r}(N/d_r)$$ --------(8.1)

Example:

$N = 216 = 2^33^3$

(1) $216$  --- $F_{216}(1) = 1$
(2) $108 \times 2$  --- $F_{108}(2) = 1$
(3) $72 \times 3$  --- $F_{72}(3) = 1$
(4) $54 \times 4$  --- $F_{54}(4) = 2$
(5) $54 \times 2 \times 2$
(6) $36 \times 6$  --- $F_{36}(6) = 2$
(7) $36 \times 3 \times 2$
(8) $27 \times 8$  --- $F_{27}(8) = 3$
(9) $27 \times 4 \times 2$
(10) $27 \times 2 \times 2 \times 2$
(11) $24 \times 9$  --- $F_{24}(9) = 2$
(12) $24 \times 3 \times 3$
(13) $18 \times 12$  --- $F_{18}(12) = 4$
(14) $18 \times 6 \times 2$
(15) $18 \times 4 \times 3$
(16) $18 \times 3 \times 2 \times 2$
(17) $12 \times 9 \times 2$  --- $F_{12}(18) = 3$
(18) $12 \times 6 \times 3$
(19) $12 \times 3 \times 3 \times 2$
(20) $9 \times 8 \times 3$  --- $F_9(24) = 5$
(21) $9 \times 6 \times 4$
(22) $9 \times 6 \times 2 \times 2$
(23) $9 \times 4 \times 3 \times 2$
(24) $9 \times 3 \times 2 \times 2$
(25) $8 \times 3 \times 3 \times 3$  --- $F_8(27) = 1$
(26) $6 \times 6 \times 6$  --- $F_6(36) = 4$
(27) $6 \times 6 \times 3 \times 2$
(28) $6 \times 4 \times 3 \times 3$
(29) $6 \times 3 \times 3 \times 2 \times 2$
(30) $4 \times 3 \times 3 \times 3 \times 2 \times 2$  --- $F_4(54) = 1$
(31) $3 \times 3 \times 3 \times 2 \times 2 \times 2$  --- $F_3(72) = 1$

$--- F_2(108) = 0$

$--- F_1(216) = 0$
\[ F'(216) = \sum_{d_r/N} F'_d(216/d_r) = 31 \]

**Remarks:** This algorithm would be quite helpful in developing a computer program for the listing of SFPs.

**REFERENCES:**


[2] "The Florentine Smarandache " Special Collection, Archives of American Mathematics, Centre for American History, University of Texas at Austin, USA."