AN INEQUALITY FOR THE
SMARANDACHE FUNCTION

by

Mihaly Bencze
2212 Săcele, Str. Harmanului 6,
Jud. Brașov, Romania

Let \( S(m) = \min\{ k \mid k \in \mathbb{N} : m \mid k!\} \) be
the Smarandache Function. In this paper we prove the following

THEOREM: \( S(\prod_{k=1}^{m} m_k) \leq \sum_{k=1}^{m} S(m_k). \)

We prove by induction. For \( m=1 \) it's true.
Let \( m=2 \), then we prove \( S(m_1), m_2) \leq S(m_1) + S(m_2). \)
We have \( m_2 \mid S(m_2)! \) and if \( r \geq S(m_2) \) then
\( S(m_2) \mid r(r-1) \cdots (r-S(m_2)+1). \)
If \( t \mid S(m_1) \) then \( t \mid r(r-1) \cdots (r-S(m_2)+1) \)
so \( m_2 \mid S(m_2) \mid S(m_2)! \) and \( (S(m_2)+1) \)
From this it results \( S(m_1, m_2) \leq S(m_1) + S(m_2). \)
We suppose they are true for \( m \), and we prove for \( m+1. \)

\[
S(\prod_{k=1}^{m+1} m_k) = S(m_1, m_2) \leq S(m_1) + S(\prod_{k=2}^{m+1} m_k) \leq S(m_1) + \sum_{k=2}^{m+1} S(m_k) = \sum_{k=1}^{m+1} S(m_k). \]

REFERENCE:

<Smarandache Function Journal> (1990), No. 1, pp. 3-17.