Abstract:

The set of $p$ different equivalence classes is $\mathbb{Z}_p = \{ [0], [1], [2], \ldots, [k], \ldots, [p-1] \}$. For convenience, we have omitted the brackets and written $k$ in place of $[k]$. Thus

$$\mathbb{Z}_p = \{ 0, 1, 2, \ldots, k \ldots, p-1 \}$$

The elements of $\mathbb{Z}_p$ can be written uniquely as $m$-adic numbers. If $r = (a_{n-1} a_{n-2} \ldots a_0)_{m}$ and $s = (b_{n-1} b_{n-2} \ldots b_0)_{m}$ be any two elements of $\mathbb{Z}_p$, then $r \Delta s$ is defined as

$$(|a_{n-1} - b_{n-1}|, |a_{n-2} - b_{n-2}|, \ldots, |a_0 - b_0|)_m$$

then $(\mathbb{Z}_p, \Delta)$ is a groupoid known as SMARANDACHE GROUPOID. If we define a binary relation $r \equiv s \Leftrightarrow r \Delta C(r) = s \Delta C(s)$, where $C(r)$ and $C(s)$ are the complements of $r$ and $s$ respectively, then we see that this relation is equivalence relation and partitions $\mathbb{Z}_p$ into some equivalence classes. The equivalence class

$$D_{\equiv}(\mathbb{Z}_p) = \{ r \in \mathbb{Z}_p : r \Delta C(r) = \text{Sup}(\mathbb{Z}_p) \}$$

is defined as $D$-form. Properties of SMARANDACHE GROUPOID and $D$-form are discussed here.

Key Words: SMARANDACHE GROUPOID, complement element and $D$-form.

1. Introduction:

Let $m$ be a positive integer greater than one. Then every positive integer $r$ can be written uniquely in the form $r = a_{n-1}m^{n-1} + a_{n-2}m^{n-2} + \ldots + a_1m + a_0$ where $n \geq 0$, $a_i$ is an integer, $0 \leq a_i \leq m$, $m$ is called the base of $r$, which is denoted by $(a_{n-1} a_{n-2} \ldots a_0)_{m}$. The absolute difference of two integers $r = (a_{n-1} a_{n-2} \ldots a_0)_{m}$ and $s = (b_{n-1} b_{n-2} \ldots b_0)_{m}$ denoted by $r \Delta s$ and defined as

$$r \Delta s = (|a_{n-1} - b_{n-1}|, |a_{n-2} - b_{n-2}|, \ldots, |a_0 - b_0|)_m$$

$$= (c_{n-1}, c_{n-2} \ldots c_0)_{m},$$

where $c_i = |a_i - b_i|$ for $i = 0, 1, 2, \ldots, n-1$.

In this operation, $r \Delta s$ is not necessarily equal to $|r - s|$ i.e. absolute difference of $r$ and $s$.

If the equivalence classes of $\mathbb{Z}_p$ are expressed as $m$-adic numbers, then $\mathbb{Z}_p$ with binary operation $\Delta$ is a groupoid, which contains some non-trivial groups. This groupoid is defined as SMARANDACHE GROUPOID. Some properties of this groupoid are established here.

2. Preliminaries:

We recall the following definitions and properties to introduce SMARANDACHE GROUPOID.
Definition 2.1 (2)

Let $p$ be a fixed integer greater than one. If $a$ and $b$ are integers such that $a-b$ is divisible by $p$, then $a$ is congruent to $b$ modulo $p$ and indicate this by writing $a \equiv b \pmod{p}$. This congruence relation is an equivalence relation on the set of all integers.

The set of $p$ different equivalence classes is $\mathbb{Z}_p = \{0, 1, 2, 3, \ldots, p-1\}$

Proposition 2.2 (1)

If $a \equiv b \pmod{p}$ and $c \equiv d \pmod{p}$

i) $a + c \equiv b + d \pmod{p}$

ii) $a \times c \equiv b \times d \pmod{p}$

Proposition 2.3 (2)

Let $m$ be a positive integer greater than one. Then every integer $r$ can be written uniquely in the form

$r = a_n m^{n-1} + a_{n-1} m^{n-2} + \ldots + a_1 m + a_0$

where $n \geq 0$, $a_i$ is an integer $0 \leq a_i < m$. Here $m$ is called the base of $r$, which is denoted by $(a_{n-1}a_{n-2} \ldots a_1a_0)_m$.

Proposition 2.4

If $r = (a_{n-1}a_{n-2} \ldots a_1a_0)_m$ and $s = (b_{n-1}b_{n-2} \ldots b_1b_0)_m$, then

i) $r = s$ if and only if $a_i = b_i$ for $i = 0, 1, 2, \ldots, n-1$.

ii) $r < s$ if and only if $(a_{n-1}a_{n-2} \ldots a_1a_0)_m < (b_{n-1}b_{n-2} \ldots b_1b_0)_m$

iii) $r > s$ if and only if $(a_{n-1}a_{n-2} \ldots a_1a_0)_m > (b_{n-1}b_{n-2} \ldots b_1b_0)_m$

3. Smarandache groupoid:

Definition 3.1

Let $r = (a_{n-1}a_{n-2} \ldots a_1a_0)_m$ and $s = (b_{n-1}b_{n-2} \ldots b_1b_0)_m$, then the absolute difference denoted by $\Delta$ of $r$ and $s$ is defined as

$r \Delta s = (c_{n-1}c_{n-2} \ldots c_1 \ldots c_0c'_0)_m$ where $c_i = |a_i - b_i|$ for $i = 0, 1, 2, \ldots, n-1$.

Here, $r \Delta s$ is not necessarily equal to $|r - s|$. For example

$5 = (101)_2$ and $6 = (110)_2$ and $5 \Delta 6 = (011)_2 = 3$ but $|5 - 6| = 1$.

In this paper, we shall consider $5 \Delta 6 = 3$, not $5 \Delta 6 = 1$.

Definition 3.2

Let $(\mathbb{Z}_p, +_p)$ be a commulative group of order $p = m^n$. If the elements of $\mathbb{Z}_p$ are
expressed as \( m \)-adic numbers as shown below:

\[
\begin{align*}
0 &= (00 \ldots \ldots 00)_m \\
1 &= (00 \ldots \ldots 01)_m \\
2 &= (00 \ldots \ldots 02)_m \\
&\quad \ldots \ldots \ldots \ldots \\
m - 1 &= (00 \ldots \ldots 0 m-1)_m \\
m &= (00 \ldots \ldots 1 0)_m \\
m + 1 &= (00 \ldots \ldots 1 1)_m \\
&\quad \ldots \ldots \ldots \ldots \\
m^2 - 1 &= (00 \ldots \ldots m-1 m-1)_m \\
m^2 &= (00 \ldots \ldots 1 0 0)_m \\
&\quad \ldots \ldots \ldots \ldots \\
m^n - 1 &= (m-1 m-1 \ldots \ldots m-1 m-1)_m \\
\end{align*}
\]

The set \( \mathbb{Z}_p \) is closed under binary operation \( \Delta \). Thus \( (\mathbb{Z}_p, \Delta) \) is a groupoid. The elements \((00 \ldots \ldots 00)_m\) and \((m-1 m-1 \ldots \ldots m-1 m-1)_m\) are called infimum and supremum of \( \mathbb{Z}_p \).

The set \( H_i \) of the elements noted below:

\[
\begin{align*}
0 &= (00 \ldots \ldots 00)_m \\
1 &= (00 \ldots \ldots 01)_m \\
m &= (00 \ldots \ldots 1 0)_m \\
m + 1 &= (00 \ldots \ldots 1 1)_m \\
&\quad \ldots \ldots \ldots \ldots \\
\end{align*}
\]

\[
\frac{m^n - m}{m - 1} = (0 1 \ldots \ldots 1 0)_m = \alpha \text{ (say)}
\]

\[
\frac{m^{n+1} - 1}{m - 1} = (0 1 \ldots \ldots 1 1)_m = \beta \text{ (say)}
\]

\[
\frac{m^n - m}{m - 1} = (1 1 \ldots \ldots 1 0)_m = \gamma \text{ (say)}
\]

\[
\frac{m^{n+1} - 1}{m - 1} = (1 1 \ldots \ldots 1 1)_m = \delta \text{ (say)}
\]

is a proper subset of \( \mathbb{Z}_p \).
is a group of order \(2^n\) and its group table is as follows:

\[
\begin{array}{cccc|cccc}
\Delta & 0 & 1 & m & m+1 & \alpha & \beta & \gamma & \delta \\
0 & 0 & 1 & m & m+1 & \alpha & \beta & \gamma & \delta \\
1 & 1 & 0 & m+1 & m & \beta & \alpha & \delta & \gamma \\
m & m & m+1 & 0 & 1 & \gamma & \delta & \alpha & \beta \\
m+1 & m+1 & m & 1 & 0 & \delta & \gamma & \beta & \alpha \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\alpha & \alpha & \beta & \gamma & \delta & \cdots & 0 & 1 & m & m+1 \\
\beta & \beta & \alpha & \delta & \gamma & \cdots & 1 & 0 & m+1 & m \\
\gamma & \gamma & \delta & \alpha & \beta & \cdots & m & m+1 & 0 & 1 \\
\delta & \delta & \gamma & \beta & \alpha & \cdots & m+1 & m & 1 & 0 \\
\end{array}
\]

Table - 1

Similarly the proper sub-sets
\[H_2 = \{0, 2, 2m, 2(m+1)\} \quad \cdots \quad 2\alpha, 2\beta, 2\gamma, 2\delta\]  
\[H_3 = \{0, 3, 3m, 3(m+1)\} \quad \cdots \quad 3\alpha, 3\beta, 3\gamma, 3\delta\]  
\[H_{m-1} = \{0, m-1, m(m-1), m^2-1\} \quad \cdots \quad (m-1)\alpha, (m-1)\beta, (m-1)\gamma, (m-1)\delta\]
are groups of order \(2^n\) under the operation absolute difference. So the groupoid
\((Zp, \Delta)\) contains mainly the groups \((H_1, \Delta), (H_2, \Delta), (H_3, \Delta) \quad \cdots \quad (H_{m-1}, \Delta)\) and this groupoid is defined as SMARANDACHE GROUPOID. Here we use S.Gd. in place of SMARANDACHE GROUPOID.

Remarks 3.2

i) Let \((Zp, +p)\) be a commutative group of order \(p\), where \(m^{n-1} < p < m^n\), then \((Zp, \Delta)\) is not groupoid.

For example \((Z_5, +5)\) is a commutative group of order 5, where \(2^2 < p < 2^3\).

Here \(Z_2 = \{0, 1, 2, 3, 4\}\) and
\[
0 = (0 0 0)_2, \quad 4 = (1 0 0)_2 \\
1 = (0 0 1)_2, \quad 5 = (1 0 1)_2 \\
2 = (0 1 0)_2, \quad 6 = (1 1 0)_2 \\
3 = (0 1 1)_2, \quad 7 = (1 1 1)_2
\]
A composition table of $\mathbb{Z}_4$ is given below:

<table>
<thead>
<tr>
<th>$\Delta$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
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<td>3</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>0</td>
</tr>
</tbody>
</table>

Table - 2

Hence $\mathbb{Z}_4$ is not closed under the operation $\Delta$, i.e. $(\mathbb{Z}_4, \Delta)$ is not a groupoid.

ii) S. Gd. is not necessarily associative.

Let $1 = (00 \ldots 01)_m$
$2 = (00 \ldots 02)_m$ and
$3 = (00 \ldots 03)_m$ be three elements of $(\mathbb{Z}_p, \Delta)$, then
\[(1 \Delta 2) \Delta 3 = 2 \quad \text{and} \quad 1 \Delta (2 \Delta 3) = 0\]
i.e. $(1 \Delta 2) \Delta 3 \neq 1 \Delta (2 \Delta 3)$.

iii) S. Gd. is commutative.

iv) S. Gd. has identity element $0 = (00 \ldots 0)_m$

v) Each element of S. Gd. is self inverse i.e. $\forall p \in \mathbb{Z}_p, \ p \Delta p = 0$.

Proposition 3.3

If $(H, \Delta)$ and $(K, \Delta)$ be two groups of order $2^n$ contained in S. Gd. $(\mathbb{Z}_p, \Delta)$, then $H$ is isomorphic to $K$.

Proof is obvious.

4. Complement element in S. Gd. $(\mathbb{Z}_p, \Delta)$.

Definition 4.1

Let $(\mathbb{Z}_p, \Delta)$ be a S. Gd., then the complement of any element $p \in \mathbb{Z}_p$ is equal to $p \Delta \text{Sup}(\mathbb{Z}_p) = p \Delta m^n - 1$ i.e. $C(p) = m^n - 1 \Delta p$. This function is known as complement function and it satisfies the following properties.

i) $C(0) = m^n - 1$

ii) $C(m^n - 1) = 0$

iii) $C(C(p)) = p$ $\forall$ $p \in \mathbb{Z}_p$

iv) If $p \leq q$ then $C(p) \geq C(q)$
Definition 4.2

An element \( p \) of a S. Gd. \( Z_p \) is said to be self complement if \( p \Delta \text{sup}(Z_p) = p \) i.e. \( C(p) = p \).

If \( m \) is an odd integer greater than one, then \( \frac{m^2 - 1}{2} \) is the self complement of \( (Z_p, \Delta) \).

If \( m \) is an even integer, then there exists no self complement in \( (Z_p, \Delta) \).

Remarks 4.3

i) The complement of an element belonging to a S. Gd. is unique.

ii) The S. Gd. is closed under complement operation.

5. A binary relation in S. Gd.

Definition 5.1

Let \( (Z_p, \Delta) \) be a S. Gd. An element \( p \) of \( Z_p \) is said to be related to \( q \in Z_p \) iff \( p \Delta C(p) = q \Delta C(q) \) and written as \( p \equiv q \Leftrightarrow p \Delta C(p) = q \Delta C(q) \).

Proposition 5.2

For the elements \( p \) and \( q \) of S. Gd. \( (Z_p, \Delta) \), \( p \equiv q \Leftrightarrow C(p) \equiv C(q) \).

Proof: By definition

\[
p \equiv q \Leftrightarrow p \Delta C(p) = q \Delta C(q).
\]

\[
\Leftrightarrow C(p) \Delta p = C(q) \Delta q
\]

\[
\Leftrightarrow C(p) \Delta C(C(p)) = C(q) \Delta C(C(q))
\]

\[
\Leftrightarrow C(p) \equiv C(q)
\]

Proposition 5.3

Let \( (Z_p, \Delta) \) be a S. Gd., then a binary relation \( p \equiv q \Leftrightarrow p \Delta C(p) = q \Delta C(q) \) for \( p, q \in Z_p \), is an equivalence relation.

Proof: Let \( (Z_p, \Delta) \) be a S. Gd. and for any two elements \( p \) and \( q \) of \( Z_p \), let us define a binary relation \( p \equiv q \Leftrightarrow p \Delta C(p) = q \Delta C(q) \).

This relation is

i) reflexive for if \( p \) is an arbitrary element of \( Z_p \), we get \( p \Delta C(p) = p \Delta C(p) \) for all \( p \in Z_p \). Hence \( p \equiv p \Leftrightarrow p \Delta C(p) = p \Delta C(p) \) \( \forall \ p \in Z_p \).

ii) symmetric, for if \( p \) and \( q \) are any elements of \( Z_p \) such that \( p \equiv q \), then \( p \equiv q \Leftrightarrow p \Delta C(p) = q \Delta C(q) \)

\[
\Leftrightarrow q \Delta C(q) = p \Delta C(p)
\]

\[
\Leftrightarrow q \equiv p
\]
iii) transitive, for $p$, $q$, $r$ are any three elements of $\mathbb{Z}_p$ such that

\[ p \equiv q \quad \text{and} \quad q \equiv r, \text{ then} \]

\[ p \equiv q \iff p \Delta C(p) = q \Delta C(q) \quad \text{and} \]

\[ q \equiv r \iff q \Delta C(q) = r \Delta C(r). \]

Thus \[ p \Delta C(p) = r \Delta C(r) \iff p \equiv r \]

Hence \[ p \equiv q \quad \text{and} \quad q \equiv r \quad \text{implies} \quad p \equiv r \]

6. D - Form of S. Gd.

Let $(\mathbb{Z}_p, \Delta)$ be a S. Gd. of order $m^n$. Then the equivalence relation referred in the proposition 5.3 partitions $\mathbb{Z}_p$ into mutually disjoint classes.

**Definition 6.1**

If $r$ be any element of S. Gd. $(\mathbb{Z}_p, \Delta)$ such that $r \Delta C(r) = x$, then the equivalence class generated by $x$ is denoted by $D_x$ and defined by

\[ D_x = \{ r \in \mathbb{Z}_p : r \Delta C(r) = x \} \]

The equivalence class generated by $\sup(\mathbb{Z}_p)$ and defined by

\[ D_{\sup \mathbb{Z}_p} = \{ r \in \mathbb{Z}_p : r \Delta C(r) = \sup(\mathbb{Z}_p) \} \]

is called the D - form of $(\mathbb{Z}_p, \Delta)$.

**Example 6.2**

Let $(\mathbb{Z}_9, +9)$ be a commutative group, then $\mathbb{Z}_9 = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8 \}$. If the elements of $\mathbb{Z}_9$ are written as 3-adic numbers, then

\[ \mathbb{Z}_9 = \{ (00), (01), (02), (10), (11), (12), (20), (21), (22) \} \]

and $(\mathbb{Z}_9, \Delta)$ is a S. Gd. of order $3^2 = 9$. Its composition table is as follows:

<table>
<thead>
<tr>
<th>$\Delta$</th>
<th>0</th>
<th>1</th>
<th>2</th>
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<th>4</th>
<th>5</th>
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<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table - 3**
Here \( 0 \Delta C(0) = 0 \Delta 8 = 8 \)
\( 1 \Delta C(1) = 1 \Delta 7 = 6 \)
\( 2 \Delta C(2) = 2 \Delta 6 = 8 \)
\( 3 \Delta C(3) = 3 \Delta 5 = 2 \)
\( 4 \Delta C(4) = 4 \Delta 4 = 0 \)
\( 5 \Delta C(5) = 5 \Delta 3 = 2 \)
\( 6 \Delta C(6) = 6 \Delta 2 = 8 \)
\( 7 \Delta C(7) = 7 \Delta 1 = 6 \)
\( 8 \Delta C(8) = 8 \Delta 0 = 8 \)

Hence \( D_g = \{ 0, 2, 6, 8 \} = \{ (00)_3, (02)_3, (20)_3, (22)_3 \} \)
\( D_6 = \{ 1, 7 \} \)
\( D_2 = \{ 3, 5 \} \)
\( D_0 = \{ 4 \} \)

The self-complement element of \((Z_p, \Delta)\) is 4 and D-form of this S. Gd. is \( \{ 0, 2, 6, 8 \} = D_g \)

Here \( Z_p = D_0 \cup D_2 \cup D_6 \cup D_g \).

Proposition 6.3
Any two equivalence classes in a S. Gd. \((Z_p, \Delta)\) are either disjoint or identical.
Proof is obvious.

Proposition 6.4
Every S. Gd. \((Z_p, \Delta)\) is equal to the union of its equivalence classes.
Proof is obvious.

Proposition 6.5
Every D-form of a S. Gd. \((Z_p, \Delta)\) is a commutative group.
Proof: Let \((Z_p, \Delta)\) be a S. Gd. of order \( P = m^n \). The elements of D-form of this groupoid are as follows.
\[
\begin{align*}
0 &= (00 \ldots 00)_{m} \\
m - 1 &= (00 \ldots 0 m^{-1})_{m} \\
m^2 - m &= (00 \ldots m^{-1} 0)_{m} \\
m^2 - 1 &= (00 \ldots m^{-1} m^{-1})_{m} \\
&\vdots \\
m^{n-1} - m &= (0 m^{-1} \ldots m^{-1} 0)_{m} \\
m^{n-1} - 1 &= (0 m^{-1} \ldots m^{-1} m^{-1})_{m} \\
m^n - m &= (m^{-1} m^{-1} \ldots m^{-1} 0)_{m} \\
m^n - 1 &= (m^{-1} m^{-1} \ldots m^{-1} m^{-1})_{m}
\end{align*}
\]
Here (D_{m^0-1}) is a commutative group and its table is given below:

<table>
<thead>
<tr>
<th>Δ</th>
<th>0</th>
<th>m-1</th>
<th>m^2-m</th>
<th>m^2-1</th>
<th>...</th>
<th>m^1-m</th>
<th>m^{1-1}</th>
<th>m^0-m</th>
<th>m^{0-1}</th>
</tr>
</thead>
<tbody>
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<td>m-1</td>
<td>m^2-m</td>
<td>m^2-1</td>
<td>...</td>
<td>m^1-m</td>
<td>m^{1-1}</td>
<td>m^0-m</td>
<td>m^{0-1}</td>
</tr>
<tr>
<td>m-1</td>
<td>m-1</td>
<td>0</td>
<td>m^2-m</td>
<td>m^2-1</td>
<td>...</td>
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<td>m^2-m</td>
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<td>m^1-m</td>
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<td></td>
</tr>
</tbody>
</table>

Table - 4

Remarks 6.6

Let (Z_p, Δ) be a S. Gd. of order m_p.

The equivalence relation p ≡ q ⇔ p Δ C(p) = q Δ C(q) partitions Z_p into some equivalence classes.

i) If m is odd integer, then the number of elements belonging to the equivalence classes are not equal. In the example 6.2, the number of elements belonging to the equivalence classes D_0, D_2, D_6, D_4 are not equal due to m = 3.

ii) If m is even integer, then the number of elements belonging to the equivalence classes are equal.

For example, Z_{16} = { 0, 1, 2, ..., 15 } be a commutative group. If the elements of Z_{16} are expressed as 4-adic numbers, then (Z_{16}, Δ) is a S. Gd. The composition table of (Z_{16}, Δ) is given below:

...
The number of elements of the equivalence classes are equal due to $m = 4$, which is even integer.

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