Five Properties of the Smarandache Double Factorial Function

Felice Russo
Via A. Infante 7
67051 Avezzano (Ag) Italy
felice.russo@katamail.com

Abstract

In this paper some properties of the Smarandache double factorial function have been analyzed.

In [1], [2], [3] and [4] the Smarandache double factorial Sdf(n) function is defined as the smallest number such that Sdf(n)!! is divisible by n, where the double factorial by definition is given by [6]:

\[ m!! = 1 \times 3 \times 5 \times \ldots \times m, \text{ if } m \text{ is odd}; \]
\[ m!! = 2 \times 4 \times 6 \times \ldots \times m, \text{ if } m \text{ is even}. \]

In [2] several properties of that function have been analyzed. In this paper five new properties are reported.

1. \( Sdf(p^{k+2}) = p^2 \) where \( p = 2 \cdot k + 1 \) is any prime and \( k \) any integer

Let's consider the prime \( p = 2k + 1 \). Then:

\[ 1 \cdot 3 \cdot 5 \cdot 7 \cdot \ldots \cdot p \cdot \ldots \cdot 3p \cdot \ldots \cdot 5p \cdot \ldots \cdot p^2 = m \cdot p^{k+2} \text{ where } m \text{ is any integer.} \]

This because the number of terms multiples of \( p \) up to \( p^2 \) are \( k+1 \) and the last term contains two times \( p \).

Then \( p^2 \) is the least value such that \( 1 \cdot 3 \cdot 5 \cdot 7 \cdot \ldots \cdot p^2 \) is divisible by \( p^{k+2} \).

2. \( Sdf(p^2) = 3 \cdot p \) where \( p \) is any odd prime.
In fact for any odd \( p \) we have:

\[
1 \cdot 3 \cdot 5 \cdot 7 \cdots \cdot p \cdots \cdot 3p = m \cdot p^2 \quad \text{where } m \text{ is any integer.}
\]

3. \( Sd \left( k \cdot \left( \frac{10^n - 1}{9} \right) \right) = Sd \left( \frac{10^n - 1}{9} \right) \) where \( n \) is any integer >1 and \( k=3,5,7,9 \)

Let's suppose that \( Sd \left( \frac{10^n - 1}{9} \right) = m \) then:

\[
1 \cdot 3 \cdot 5 \cdot 7 \cdots \cdot m = a' \cdot \left( \frac{10^n - 1}{9} \right) \quad \text{where } a' \text{ is any integer. But in the previous multiplication there are factors multiple of 3,5,7 and 9 and then:}
\]

\[
1 \cdot 3 \cdot 5 \cdot 7 \cdots \cdot m = a' \cdot k \cdot \left( \frac{10^n - 1}{9} \right) \quad \text{where } a' \text{ is any integer and } k=3,5,7,9. \text{ Then:}
\]

\[
Sd \left( k \cdot \left( \frac{10^n - 1}{9} \right) \right) = m = Sd \left( \frac{10^n - 1}{9} \right)
\]

4. \( Sd \left( k \cdot \left( \frac{10^n - 1}{9} \right) \right) = Sd \left( 2 \cdot \left( \frac{10^n - 1}{9} \right) \right) \) where \( n \) is any integer >1 and \( k=2,4,6,8 \)

Let's suppose that \( Sd \left( 2 \cdot \left( \frac{10^n - 1}{9} \right) \right) = m \) then:

\[
2 \cdot 4 \cdot 6 \cdot 8 \cdots \cdot m = a \cdot 2 \cdot \left( \frac{10^n - 1}{9} \right) \quad \text{where } a \text{ is any integer. But in the previous} 
\]
multiplication there are factors multiple of 4, 6 and 8 and then:

\[ 2 \cdot 4 \cdot 6 \cdot 8 \cdot \ldots \cdot m = a' \cdot 2 \cdot k \cdot \left( \frac{10^n - 1}{9} \right) \]

where \( a' \) is any integer and \( k = 4, 6, 8 \).

Then:

\[ Sdf \left( k \cdot \left( \frac{10^n - 1}{9} \right) \right) = m = Sdf \left( 2 \cdot \left( \frac{10^n - 1}{9} \right) \right) \]

5. \( Sdf \left( p^n \right) = (2 \cdot m - 1) \cdot p \) for \( p \equiv (2m - 1) \). Here \( m \) is any integer and \( p \) any odd prime.

This is a generalization of property number 2 reported above.

**References.**


http://www.gallup.unm.edu/~smarandache/SNAOINT.txt


