Introducing the SMARANDACHE-KUREPA
and SMARANDACHE-WAGSTAFF Functions

by

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Definition A.

The left-factorial function is defined by D. Kurepa thus:

\[ !n = 0! + 1! + 2! + 3! + \ldots + (n-1)! \]

whilst S. S. Wagstaff prefers:

\[ B_n = !(n+1) - 1 = 1! + 2! + 3! + \ldots + n! \]

The following properties should be observed:

(i) \( !n \) is only divisible by \( n \) when \( n = 2 \).

(ii) 3 is a factor of \( B_n \) if \( n \) is greater than 1.

(iii) 9 is a factor of \( B_n \) if \( n \) is greater than 4.

(iv) 99 is a factor of \( B_n \) if \( n \) is greater than 9.

There are no other such cases of divisibility of \( B_n \) for \( n \) less than a thousand.

The tabulated values of these two functions together with their prime factors begin:

\[
\begin{array}{lll}
\text{n} & !n & B_n \\
1 & 1 & 1 \\
2 & 2 & 3 \\
3 & 4=2\cdot2 & 9=3\cdot3 \\
4 & 10=2\cdot5 & 33=3\cdot11 \\
5 & 34=2\cdot17 & 153=3\cdot3\cdot17 \\
6 & 154=2\cdot7\cdot11 & 873=3\cdot3\cdot97 \\
7 & 8742=19\cdot23 & 5913=3\cdot3\cdot3\cdot73 \\
8 & 5914=2\cdot2957 & 46233=3\cdot3\cdot11\cdot467 \\
9 & 46234=2\cdot23117 & 409113=3\cdot3\cdot131\cdot347 \\
10 & 409114=2\cdot204557 & \\
\end{array}
\]

TABLE I.
"Intuitive Thought": There appear to be a disproportionate (unexpectedly high) number of large primes in this table?

**Definition B.**

For prime $p$ not equal to 3 define the SMARANDACHE-KUREPA Function, $SK(p)$, as the smallest integer such that $!SK(p)$ is divisible by $p$.
For prime $p$ not equal to 2 or 5 define the SMARANDACHE-WAGSTAFF Function, $SW(p)$, as the smallest integer such that $B_{sw(p)}$ is divisible by $p$.

The tabulation of these two functions begins:

<table>
<thead>
<tr>
<th>$p$</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>11</th>
<th>13</th>
<th>17</th>
<th>19</th>
<th>23</th>
<th>131</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SK(p)$</td>
<td>2</td>
<td>*</td>
<td>4</td>
<td>6</td>
<td>6</td>
<td>?</td>
<td>5</td>
<td>7</td>
<td>7</td>
<td>?</td>
</tr>
</tbody>
</table>

Where the entry * denotes that the value is not defined and the entry ? denotes not available from TABLE I above.

Some unanswered questions:

1. Are there other (*) - entries i.e. undefined values in the above table.
2. What is the distribution function of integers in both $SK(p)$, $SW(p)$ and their union?
3. When, in general, is $SK(p) = SW(p)$?

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