LENGTH / EXTENT OF SMARANDACHE FACTOR PARTITIONS

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ABSTRACT: In [1] we define SMARANDACHE FACTOR PARTITION FUNCTION (SFP), as follows:

Let \( \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_r \) be a set of \( r \) natural numbers and \( p_1, p_2, p_3, \ldots, p_r \) be arbitrarily chosen distinct primes then \( F(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_r) \) called the Smarandache Factor Partition of \( (\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_r) \) is defined as the number of ways in which the number

\[
N = \frac{\alpha_1 \alpha_2 \alpha_3 \ldots \alpha_r}{p_1 p_2 p_3 \ldots p_r}
\]

could be expressed as the product of its' divisors. For simplicity, we denote \( F(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_r) = F'(N) \), where

\[
N = \frac{\alpha_1 \alpha_2 \alpha_3 \ldots \alpha_r}{p_1 p_2 p_3 \ldots p_r \ldots p_n}
\]

and \( p_r \) is the \( r^{th} \) prime. \( p_1 = 2, p_2 = 3 \) etc.

Also for the case

\[
\alpha_1 = \alpha_2 = \alpha_3 = \ldots = \alpha_r = \ldots = \alpha_n = 1
\]

we denote

\[
F(1, 1, 1, 1, 1 \ldots) = F(1^n)
\]

In the present note we define two interesting parameters the
length and extent of an SFP and study the interesting properties they exhibit for square free numbers.

**DISCUSSION:**

**DEFINITION:** Let $F'(N) = R$

**LENGTH:** If we denote each SFP of $N$, say like $F_1, F_2, \ldots, F_R$ arbitrarily and let $F_k$ be the SFP representation of $N$ as the product of its divisors as follows:

$F_k \rightarrow \ N = (h_1)(h_2)(h_3)(h_4) \ldots (h_t)$, where each $h_i \ (1 < i < t)$ is an entity in the SFP $\ 'F_k' \ of \ N$. Then $T(F_k) = t$ is defined as the 'length' of the factor partition $F_k$.

e.g. say $60 = 15 \times 2 \times 2$, is a factor partition $F_k$ of $60$. Then

$T(F_k) = 3$.

$T(F_k)$ can also be defined as one more than the number of product signs in the factor partition.

**EXTENT:** The extent of a number is defined as the sum of the lengths of all the SFPs.

Consider $F(1#3)$

$N = p_1p_2p_3 = 2 \times 3 \times 5 = 30$.

<table>
<thead>
<tr>
<th>SN</th>
<th>Factor Partition</th>
<th>length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>15 X 2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>10 X 3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>6 X 5</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>5 X 3 X 2</td>
<td>3</td>
</tr>
</tbody>
</table>

Extent (30) = $\sum$ length = 10

We observe that
Consider $F(1#4)$

$N = 2 \times 3 \times 5 \times 7 = 210$

<table>
<thead>
<tr>
<th>SN</th>
<th>Factor Partition</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>210</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>105 X 2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>70 X 3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>42 X 5</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>35 X 6</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>35 X 3 X 2</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>30 X 7</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>21 X 10</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>21 X 5 X 2</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>15 X 14</td>
<td>2</td>
</tr>
<tr>
<td>11</td>
<td>15 X 7 X 2</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>14 X 5 X 2</td>
<td>3</td>
</tr>
<tr>
<td>13</td>
<td>10 X 7 X 3</td>
<td>3</td>
</tr>
<tr>
<td>14</td>
<td>7 X 6 X 5</td>
<td>3</td>
</tr>
<tr>
<td>15</td>
<td>7 X 5 X 3 X 2</td>
<td>4</td>
</tr>
</tbody>
</table>

Extent(210) = $\sum$ length = 37

We observe that

$F(1#5) - F(1#4) = 37 = $Extent \{ F(1#4) \}$

Similarly it has been verified that

$F(1#6) - F(1#5) = $Extent \{ F(1#5) \}$

**CONJECTURE (6.1)**

$F(1#(n+1)) - F(1#n) = $Extent \{ F(1#n) \}$

**CONJECTURE (6.2)**

$F(1#(n+1)) = \sum_{r=0}^{n} $Extent \{ F(1#r) \}$

Motivation for this conjecture:
If conjecture (1) is true then we would have
\[ F(1#2) - F(1#1) = \text{Extent} \{ F(1#1) \} \]
\[ F(1#3) - F(1#2) = \text{Extent} \{ F(1#2) \} \]
\[ F(1#4) - F(1#3) = \text{Extent} \{ F(1#3) \} \]
\[ \vdots \]
\[ F(1#(n+1)) - F(1#n) = \text{Extent} \{ F(1#n) \} \]

Summing up we would get
\[ F(1#(n+1)) - F(1#1) = \sum_{r=1}^{n} \text{Extent} \{ F(1#r) \} \]
F(1#1) = 1 = Extent (F(1#0) can be taken, hence we get
\[ F(1#(n+1)) = \sum_{r=0}^{n} \text{Extent} \{ F(1#r) \} \]

Another Interesting Observation:

Given below is the chart of \( r \) versus \( w \) where \( w \) is the number of

SFPs having same length \( r \).

\[ F(1#0) = 1 \], \( \sum r \cdot w = 1 \)
\[ \begin{array}{c|c}
 r & 1 \\
 w & 1 \\
\end{array} \]

\[ F(1#1) = 1 \], \( \sum r \cdot w = 1 \)
\[ \begin{array}{c|c}
 r & 1 \\
 w & 1 \\
\end{array} \]

\[ F(1#2) = 2 \], \( \sum r \cdot w = 3 \)
\[ \begin{array}{c|c|c}
 r & 1 & 2 \\
 w & 1 & 1 \\
\end{array} \]

\[ F(1#3) = 5 \], \( \sum r \cdot w = 10 \)
\[ \begin{array}{c|c|c|c}
 r & 1 & 2 & 3 \\
 w & 1 & 3 & 1 \\
\end{array} \]
The interesting observation is the row of \( w \) is the same as the \( n^{th} \) row of the SMARANDACHE STAR TRIANGLE. (ref.: [4])

**CONJECTURE (6.3)**

\[
W_r = a_{(n,r)} = \frac{1}{r!} \sum_{k=0}^{r} (-1)^{r-k} \cdot C_k \cdot k^n
\]

where \( W_r \) is the number of SFPs of \( F(1\#n) \) having length \( r \).

**Further Scope:** One can study the length and contents of other cases (other than the square-free numbers.) explore for patterns if any.

**REFERENCES:**


[4] "The Florentine Smarandache " Special Collection, Archives of American Mathematics, Centre for American History, University of Texax at Austin, USA.