New Prime Numbers

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I have found some new prime numbers using the PROTH program of Yves Gallot. This program is based on the following theorem:

**Proth Theorem (1878):**
Let $N = k \cdot 2^n + 1$ where $k < 2^n$. If there is an integer number $a$ so that

$$\frac{N-1}{a^2} \equiv -1 \pmod{N}$$

then $N$ is prime.

The Proth program is a test for primality of greater numbers defined as $k \cdot b^n + 1$ or $k \cdot b^n - 1$. The program is made to look for numbers of less than 5,000,000 digits and it is optimized for numbers of more than 1,000 digits.

Using this program, I have found the following prime numbers:

- $3239 \cdot 2^{12345} + 1$ with 3720 digits $a = 3, a = 7$
- $7551 \cdot 2^{12345} + 1$ with 3721 digits $a = 5, a = 7$
- $7595 \cdot 2^{12345} + 1$ with 3721 digits $a = 3, a = 11$
- $9363 \cdot 2^{12321} + 1$ with 3713 digits $a = 5, a = 7$

Since the exponents of the first three numbers are Smarandache number $\text{Sm}(5) = 12345$ we can call this type of prime numbers, prime numbers of Smarandache.

Helped by the MATHEMATICA program, I have also found new prime numbers which are a variant of prime numbers of Fermat. They are the following:

$$2^n \cdot 3^2 - 2^n - 3^2$$

for $n = 1, 4, 5, 7$.

It is important to mention that for $n = 7$ the number which is obtained has 100 digits.

Chris Nash has verified the values $n = 8$ to $n = 20$, this last one being a number of 815,951 digits, obtaining that they are all composite. All of them have a tiny factor except $n = 13$. 
References:

www.gallup.unm.edu/~Smarandache.
The Prime Pages. www.utm.edu/research/primes.

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