Let \( p_n \) denote the \( n \)-th prime number. One of Smarandache's conjectures in [3] is the following inequality:

\[
p_{n+1}/p_n \leq 5/3, \text{ with equality for } n = 2. \tag{1}
\]

Clearly, for \( n = 1, 2, 3, 4 \) this is true, and for \( n = 2 \) there is equality. Let \( n > 4 \). Then we prove that (1) holds true with strict inequality. Indeed, by a result of Dressler, Pigno and Young (see [1] or [2]) we have

\[
p_{n+1}^2 \leq 2p_n^2. \tag{2}
\]

Thus \( p_{n+1}/p_n \leq \sqrt{2} \leq 5/3, \) since \( 3\sqrt{2} < 5 \) (i.e. 18 < 25).

This finishes the proof of (1).

References:

