On a problem concerning the Smarandache friendly prime pairs

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Abstract

In this paper a question posed in [1] and concerning the Smarandache friendly prime pairs is analysed.

Introduction

In [1] the Smarandache friendly prime pairs are defined as those prime pairs (p,q) such that:

\[ \sum_{x=p}^{q} x = p \cdot q \]  \hspace{1cm} (1)

where x denote the primes between p and q. In other words the Smarandache friendly prime pairs are the pairs (p,q) such that the sum of the primes between p and q is equal to the product of p and q.

As example let's consider the pair (2,5). In this case 2 + 3 + 5 = 2 \cdot 5 and then 2 and 5 are friendly primes. The other three pairs given in the mentioned paper are: (3,13), (5,31) and (7,53). Then the following open questions have been posed:

Are there infinitely many friendly prime pairs?

Is there for every prime p a prime q such that (p,q) is a Smarandache friendly prime pair?

In this paper we analyse the last question and a shortcut to explore the first conjecture is reported.
Results

First of all let's analyse the case \( p=11 \). Let's indicate:

\[
f(11, q) = \sum_{x=11}^{q} x \quad \text{and} \quad g(11, q) = 11 \cdot q
\]

where \( x \) denotes always the primes between 11 and \( q \).

A computer program with Ubasic software package has been written to calculate the difference between \( g(11, q) \) and \( f(11, q) \) for the 164 primes \( q \) subsequent to 11. Here below the trend of that difference.

![Graph 1](image1)

![Graph 2](image2)
As we can see the difference starts to increase, arrives to a maximum and then starts to decrease and once pass the x axis decrease in average linearly. The same thing is true for all the other primes p.

So for every prime p the search of its friend q can be performed up to:

\[ g(p, q) - f(p, q) \leq -M \]

where M is a positive constant.

For the first 1000 primes M has been choosen equal to \(10^5\).

No further friendly prime pair besides those reported in [1] has been found. According to those experimental results we are enough confident to pose the following conjecture:

*Not all the primes have a friend, that is there are prime p such that there isn't a prime q such that the (1) is true.*

Moreover a furter check of friendly prime pairs for all primes larger than 1000 and smaller than 10000 has been performed selecting M=1000000.

No further friendly prime pair has been found. Those results seem to point out that the number of friendly prime pairs is finite.

**Question:**

*Are (2,5), (3,13), (5,31) and (7,53) the only Smarandache friendly prime pairs?*

**References.**