On a problem concerning the Smarandache Unary sequence

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Abstract

In this paper a problem posed in [1] and concerning the number of primes in the Smarandache Unary sequence is analysed.

Introduction

In [1] the Smarandache Unary sequence is defined as the sequence obtained concatenating \( p_n \) digits of 1, where \( p_n \) is the n-th prime number:

\[
11, 111, 1111, 11111, 111111, 1111111, 11111111, 111111111, \ldots
\]

In the same paper the following open question is reported:

How many terms in the Smarandache Unary sequence are prime numbers?

In this paper we analyse that question and a conjecture on the number of primes belonging to the Smarandache Unary sequence is formulated.

Results

A computer program with Ubasic software package has been written to check the first 311 terms of the Unary sequence; we have found only five prime numbers. If we indicate the n-th term of the unary sequence as:

\[
u(n) = \frac{10^{p_n} - 1}{9}
\]

where \( p_n \) is the n-th prime.
those five primes have been found for $p_n$ equal to 2, 19, 23, 317 and 1031.

This means a percentage of $\frac{5}{311} \approx 1.6\%$ prime numbers. According to this experimental evidence the following conjecture can be formulated:

**Conjecture:** The number of primes in the Smarandache Unary sequence is upper limited.

**Unsolved question:** Find that upper limit.

**References.**