ON SMARANDACHE ALGEBRAIC STRUCTURES,
II: THE SMARANDACHE SEMIGROUP

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Abstract. In this paper we prove that $A(a,n)$ is a Smarandache semigroup.

Key words. Smarandache algorithm, Smarandache semigroup.

Let $G$ be a semigroup. If $G$ contains a proper subset which is a group under the same operation, then $G$ is called a Smarandache semigroup (see [2]). For example, $G = \{18, 18^2, 18^3, 18^4, 18^5\} \pmod{60}$ is a commutative multiplicative semigroup. Since the subset $\{18^2, 18^3, 18^4, 18^5\} \pmod{60}$ is a group, $G$ is a Smarandache semigroup.

Let $a, n$ be integers such that $a \neq 0$ and $n > 1$. Further, let $A(a,n)$ be defined as in [1]. In this paper we prove the following result.

Theorem. $A(a,n)$ is a Smarandache semigroup.

Proof. Under the definitions and notations of [1], let $A'(a,n) = \{a^e, a^{e+1}, \ldots, a^{e+f-1}\} \pmod{n}$. Then $A'(n,a)$ is a proper subset of $A(a,n)$.

If $e = 0$, then $a^e = 1 \in A'(a,n)$. Clear, 1 is the unit of $A'(a,n)$. Moreover, for any $a^i \in A(a,n)$ with $i > 0$, $a^{fi}$ is the inverse element of $a^i$ in $A'(a,n)$.

If $e > 0$, then $a^f \in A'(a,n)$. Since $a^f \equiv 1 \pmod{m}$, $a^f$ is the unit of $A'(a,n)$ and $a^{fi}$ is the inverse element of $a^i$ in $A'(a,n)$, where $t$ is the integer satisfying $e < ft-i \leq e+f-1$. Thus, under the Smarandache algorithm, $A'(a,n)$ is a group. It implies that $A(a,n)$ is a Smarandache
semigroup. The theorem is proved.

References


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