ON SMARANDACHE CONCATENATED SEQUENCES I: PRIME POWER SEQUENCES

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Abstract. Let \( A = \{p^n\}_{n=1}^{\infty} \), where \( p \) is a prime. Let \( C(A) = \{c_n\} \) denote the Smarandache concatenated sequence of \( A \). In this paper we prove that if \( n > 1 \) and \( p \neq 2 \) or 5, then \( c_n \) does not belong to \( A \).

Let \( A = \{a_i\}_{i=1}^{\infty} \) be an infinite increasing sequence of positive integers. For any positive integer \( n \), let \( c_n \) be the decimal integer such that

\[
(1) \quad c_n = a_1 \cdot a_2 \ldots a_n.
\]

Then sequence \( C(A) = \{c_n\}_{n=1}^{\infty} \) is called the Smarandache concatenated sequence of \( A \). In [1], Marimutha posed a general question as follows:

Question. How many terms of \( C(A) \) belong to \( A \)?

In this serial paper, we shall consider some interesting cases for the above question. In this part we prove the following result.

Theorem. Let \( A = \{p^n\}_{n=1}^{\infty} \), where \( p \) is a prime. If \( n > 1 \) and \( p \neq 2 \) or 5, then \( c_n \) does not belong to \( A \).

Proof. For any positive integer \( a \), let \( d(a) \) denote the figure number of \( a \) in the decimal system.

If \( A = \{p^n\}_{n=1}^{\infty} \), then from (1) we get

\[
2) \quad c_n = p^n + p^{n-2} \cdot 10^2 + \ldots + p^2 \cdot 10^2 + \ldots + p \cdot 10^2 + \ldots + 10^2.
\]

Further, if \( c_n \) belongs to \( A \), then we have

\[
(3) \quad c_n = p^m,
\]

where \( m \) is a positive integer with \( m > n \). It implies that

\[
(4) \quad p^2 | c_n,
\]

if \( n > 1 \). However, if \( p \neq 2 \) or 5, then \( p \neq 10^2 \) for any positive
integer $k$. Therefore, by (2), we get

\[ p^2 \mid c_n, \]

which contradicts (4). Thus, $c_n$ does not belong to $A$ in this case. The theorem is proved.

Reference