ON SMARANDACHE DIVISOR PRODUCTS

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Abstract. In this paper we give a formula for Smarandache divisor products.

Let $n$ be a positive integer. In [1, Notion 20], the product of all positive divisors of $n$ is called the Smarandache divisor product of $n$ and denoted by $P_d(n)$. In this paper we give a formula of $P_d(n)$ as follows:

$$r_1 \ldots r_k$$

Theorem. Let $n = p_1 \ldots p_k$ be the factorization of $n$, and let

$$(1) \quad r(n) = \left\{ \begin{array}{ll}
\frac{1}{2} (r_1 + 1) \ldots (r_k + 1), & \text{if } n \text{ is not a square}, \\
\frac{1}{2} ((r_1 + 1) \ldots (r_k + 1) - 1), & \text{if } n \text{ is a square}.
\end{array} \right.$$ 

Then we have $P_d(n) = n^{r(n)}$.

Proof. Let $f(n)$ denote the number of distinct positive divisors of $n$. It is a well known fact that

$$(2) \quad f(n) = (r_1 + 1) \ldots (r_k + 1),$$
(See [2, Theorem 273]). If \( n \) is not a square and \( d \) is a positive divisor of \( n \), then \( n/d \) is also a positive divisor of \( n \) with \( n / d \neq d \). It implies that

\[
\text{(3)} \quad P_d(n) = n^{f(n)/2}.
\]

Hence, by (1), (2) and (3), we get

\[
P_d(n) = n^{r(n)}.
\]

If \( n \) is a square and \( d \) is a positive divisor of \( n \) with \( d \neq \sqrt{n} \), then \( n/d \) is also a positive divisor of \( n \) with \( n / d \neq d \). So we have

\[
\text{(4)} \quad P_d(n) = \frac{n^{f(n)/2}}{\sqrt{n}} = n^{f(n+1)/2}.
\]

Therefore, by (1), (2) and (4), we get

\[
P_d(n) = n^{r(n)} \text{ too.}
\]

The theorem is proved.

References.