ON SMARANDACHE GENERAL CONTINUED FRACTIONS

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Abstract. Let $A = \{a_n\}_{n=1}^\infty$ and $B = \{b_n\}_{n=1}^\infty$ be two Smarandache type sequences. In this paper we prove that if $a_{n+1} \geq b_n > 0$ and $b_{n+1} \geq b_n$ for any positive integer $n$, the continued fraction

$$\frac{b_1}{a_1 + \frac{b_2}{a_2 + \frac{b_3}{a_3 + \cdots}}}$$

is convergent.

Let $A = \{a_n\}_{n=1}^\infty$ and $B = \{b_n\}_{n=1}^\infty$ be two Smarandache type sequences. Then the continued fraction

$$\frac{b_1}{a_1 + \frac{b_2}{a_2 + \frac{b_3}{a_3 + \cdots}}}$$

is called a Smarandache general continued fraction associated with $A$ and $B$ (see [1]). By using Roger's symbol, the continued fraction (1) can be written as

$$\frac{b_1}{a_1 + \frac{b_2}{a_2 + \frac{b_3}{a_3 + \cdots}}}$$

Recently, Castillo [1] posed the following question:

1. Question. Is the continued fractions $1 + \frac{21}{12 + \frac{321}{123 + \frac{1234}{\cdots}}}$ convergent?
In this paper we prove a general result as follows.

Theorem. If \( a_{n+1} > b_n > 0 \) and \( b_{n+1} > b_n \) for any positive integer \( n \), then the continued fraction (2) is convergent.

Proof. It is a well known fact that (2) is equal to the simple continued fraction

\[
\frac{1}{a_1 + \frac{1}{c_1 + \frac{1}{c_2 + \cdots}}},
\]

where

\[
c_{2t-1} = \frac{b_2 b_4 \ldots b_{2t-2}}{b_1 b_3 \ldots b_{2t-1}} a_{2t},
\]

and

\[
c_{2t} = \frac{b_1 b_3 \ldots b_{2t-1}}{b_2 b_4 \ldots b_{2t}} a_{2t-1}, \quad t = 1, 2, \ldots.
\]

Since \( a_{n+1} > b_n > 0 \) and \( b_{n+1} > b_n \) for any positive \( n \), we see from (4) that \( c_n \geq 1 \) for any \( n \). It implies that the simple continued fraction (3) is convergent. Thus, the Smarandache general continued fraction (2) is convergent too. The theorem is proved.

Reference