# ON SMARANDACHE REPUNIT N NUMBERS 

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#### Abstract

We define three new sets of numbers somewhat similar to Repunit numbers [1] and the Smarandache Unary numbers[2], which we call Smarandache Repunit N (SRN) numbers. We report primes, properties, conjectures and open questions concerning SRN numbers. Some subsidiary sequences are given along the way.


Reason's last step is the recognition that there are an infinite number of things which are beyond it. - Blaise Pascal, Pensees. 1670.

## 1. Introduction

In 1966 A. H. Beiler coined the term "repunit" for numbers consisting of N copies of the digit 1 . The term repunit comes from the words "repeated" and "unit". Beiler also gave the first table of known factors of repunits [1]. These numbers have the form

$$
\mathrm{R}_{\mathrm{n}}=10^{\mathrm{n}}-1 / 9
$$

It is still an unsolved problem as to whether there are infinitely many primes in $R_{n}$, and much computer time has been expended looking for repunit primes as well as factors. For example, In 1986 Williams and Dubner proved $\mathrm{R}_{1031}$ to be prime [7]. In 1999 the search was extended by Dubner who found the probable prime $\mathrm{R}_{49801}[8]$, and L . Baxter later discovered the probable prime $\mathrm{R}_{86453}$ [6]. Concerning factors of repunits, Andy Steward currently maintains a project to collate all known data on factorizations of generalized repunits [9], which have the form

$$
\mathrm{GR}_{\mathrm{n}}^{\mathrm{b}}=\mathrm{b}^{\mathrm{n}}-1 / \mathrm{b}-1
$$

In this paper we consider three new classes of numbers based on repunits, and similar to Smarandache Unary numbers (Smarandache Unary numbers are formed by repeating the digit $1 p_{n}$ times, where $p_{n}$ is the $n$-th prime), which we call Smarandache Repunit N numbers. Now for some definitions.

Definition: Smarandache Repunit Ending N numbers (SRE) (A075842) [4] are defined as $R_{n} N$ where $R_{n}$ is the nth Repunit number with $N$ concatenated to the end; or n l's followed by n . These have the form
$\operatorname{SRE}=\left(10^{\mathrm{n}}-1\right) / 9^{*} 10^{\mathrm{L}}+\mathrm{n}$, where L is the number of decimal digits of $n$.
$11,112,1113,11114,111115,1111116,11111117,111111118$, 1111111119, 111111111110, 1111111111111, 11111111111112, 111111111111113, ...

Definition: Smarandache Repunit Beginning N numbers (SRB) (A075858) [4] are defined as $N \_R_{n}$ where $R_{n}$ is the nth Repunit number with $N$ concatenated to the beginning, or $n$ followed by $n 1$ 's. These have the form

$$
\mathrm{SRB}=\mathrm{n}^{*} 10^{\mathrm{n}}+\left(10^{\mathrm{n}}-1\right) / 9 .
$$

11, 211, 3111, 41111, 511111, 6111111, 711111111, 811111111, 9111111111, 101111111111,1111111111111, 121111111111111, 1311111111111111,...

Definition: Smarandache Beginning and Ending N numbers (SRBE) (A075859) [4] are defined as $\mathrm{N}_{-} \mathrm{R}_{\mathrm{n}} \mathrm{N}$, where N is concatenated to the beginning and end of the $n$-th repunit number. These have the form

SRBE $=\mathrm{n}^{*} 10^{(\mathrm{n}+\mathrm{L})}+10^{\mathrm{L}} * \mathrm{R}+\mathrm{n}$, where R is the n -th repunit and L is the number of decimal digits of $n$.

111, 2112, 31113, 411114, 5111115, 61111116, 711111117, 8111111118, $91111111119,10111111111110,111111111111111$, 1211111111111112, ...

In this paper we consider the problem of determining which values in all three classes of Smarandache Repunit N numbers are prime, give some other properties of these numbers, make conjectures, and offer some open questions.

## 2. Prime Smarandache Repunit N Numbers

2.1 The known values of $n$ such that $\operatorname{SRE}(\mathrm{n})$ is prime (A070746) [4] are:

$$
1,7,709,2203,4841,
$$

Using PARI/GP [3] and the primality proving program Primo [5], SRE(709) was found and certified prime by the author. The probable prime SRE(2203) was also found by the author and Rick L. Shepherd found the probable prime SRE(4841). Regarding the author's computer search, it consisted mainly of brute force with a couple of simple modular arguments to weed out the numbers which were obviously not prime.

Conjecture: There are infinitely many SRE primes.
2.2 The known values of $n$ such that $\operatorname{SRB}(n)$ is prime (A068817) [4] are: $1,2,5,7,10,16,20,65,91,119,169,290,428,610,905,1051$, 3488, 4526, 6445,

Using Chris Nash's primality proving program Prime Form [10], the probable primes $\operatorname{SRB}(4526)$ and $\operatorname{SRB}(6445)$ were found by the author. We are unaware of how many of the values in the above list have actually been certified prime. Regarding the author's computer search, it consisted mainly of brute force.

Conjecture: There are infinitely many SRB primes.
2.3 Concerning SRBE primes, there are none.

Proof: Obviously the digital sum of every SRBE number is a multiple of three; this follows from their definition. And since it is a well known fact that if the digital sum of a number is divisible by three, then the number is as well. Hence, there are no SRBE primes.

## 3. Other Properties of Smarandache Repunit N Numbers and Related Quesitons

### 3.1 SRE Numbers.

Concerning squares in SRE numbers, none were found up to $\operatorname{SRE}(10000)$. Heuristically, it seems highly unlikely that there will ever be a square SRE number. A program was written in PARI/GP [3] to search for the least square with $n$ consecutive 1 's and none out of the eight squares found came close to exhibiting the required digit pattern of SRE numbers.

Conjecture: There are no square SRE numbers.
Open question: Are there any SRE cubes or higher powers?
Some values $n$ such that SRE is divisible by the sum of its digits are: $2,6,44,51,165,692,1286$, and 4884.

Open question: Are there infinitely many SRE numbers with the above property?

Some values $n$ such that the sum and product of the digits of SRE numbers (and SRB numbers) are both prime are:

$$
13,71,1112,1115,1171,1711,5111
$$

Open question: Are there infinitely many SRE numbers with the above property?

### 3.2 SRB Numbers

Concerning squares in SRB numbers, there are none, and the proof is simple.
Proof: All squares greater than 9 must terminate in one of the following two digit endings:

$$
\begin{aligned}
& 00,01,04,09,16,21,24,25,29,36,41 \\
& 44,49,56,61,64,69,76,81,84,89,96 .
\end{aligned}
$$

Obviously no SRB number will be a square, since by definition all SRB's terminate with the digits ' 11 '.

Open question: Are there any SRB cubes or higher powers?
Some values $n$ such that SRB is divisible by the sum of its digits are:

$$
33,659,2037,5021 .
$$

Open question: Are there infinitely many SRB numbers with the above property?

### 3.3 SRBE Numbers

Concerning square SRBE numbers, none were found up to SRBE(10000). It seems unlikely that there will be any SRBE squares, but the proof seems difficult. The same empirical evidence given above for the nonexistence of square SRB numbers applies to square SRBE numbers as well.

Conjecture: There are no square SRBE numbers.
Open question: Are there any SRBE cubes or higher powers?
Digression: Notice that if we divide the SRBE number 31113 by the Product of its digits we get $31113 / 9=3457$, a prime.

Open question: Are there infinitely many SRBE numbers with the above property?

## References

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