ON THE SMARANDACHE N-ARY SIEVE

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Abstract. Let $n$ be a positive integer with $n > 1$. In this paper we prove that the remaining sequence of Smarandache n-ary sieve contains infinitely many composite numbers.

Let $n$ be a positive integer with $n > 1$. Let $S_n$ denote the sequence of Smarandache n-ary sieve (see [1, Notions 29-31]). For example:

$S_2 = \{1, 3, 5, 9, 11, 13, 17, 21, 25, 27, ... \}$,

$S_3 = \{1, 2, 4, 5, 7, 8, 10, 11, 14, 16, 17, 19, 20, ... \}$

In [1], Dumitrescu and Seleacu conjectured that $S_n$ contains infinitely many composite numbers. In this paper we verify the above conjecture as follows:

Theorem. For any positive integer $n$ with $n > 1$,

$S_n$ contains infinitely many composite numbers.

Proof. By the definition of Smarandache n-ary sieve
(see [1, Notions 29-31]), the sequence $S_n$ contains the numbers $n^k + 1$ for any positive integer $k$. If $k$ is an odd integer with $k > 1$, then we have

$$n^k + 1 = (n+1)(n^{k-1} - n^{k-2} + ... + 1).$$

We see from (1) that $(n + 1)| (n^k + 1)$ and $n^k + 1$ is a composite number. Notice that there exist infinitely many odd integers $k$ with $k > 1$. Thus, $S_n$ contains infinitely many composite numbers $n^k + 1$. The theorem is proved.

References.