On three problems concerning the Smarandache LCM sequence

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Abstract

In this paper three problems posed in [1] and concerning the Smarandache LCM sequence have been analysed.

Introduction

In [1] the Smarandache LCM sequence is defined as the least common multiple (LCM) of (1,2,3,...,n):

1, 2, 6, 12, 60, 60, 420, 840, 2520, 2520, 27720, 27720, 360360, 360360, 360360, 720720 .......

In the same paper the following three problems are reported:

1. If \( a(n) \) is the \( n \)-th term of the Smarandache LCM sequence, how many terms in the new sequence obtained taking \( a(n) + 1 \) are prime numbers?

2. Evaluate \( \lim_{n \to \infty} \sum_{n} \frac{a(n)}{n!} \) where \( a(n) \) is the \( n \)-th term of the Smarandache LCM sequence

3. Evaluate \( \lim_{n \to \infty} \sum_{n} \frac{1}{a(n)} \) where \( a(n) \) is the \( n \)-th term of the Smarandache LCM sequence

In this paper we analyse those three questions.
Results

Problem 1.

Thanks to a computer program written with Ubasic software package the first 50 terms of sequence \( a(n) + 1 \), where \( a(n) \) is the \( n \)-th term of Smarandache LCM sequence, have been checked. Only 10 primes have been found excluding the repeating terms. In the following the sequence of values of \( n \leq 50 \) such that \( a(n) + 1 \) is prime is reported:

\[ \{2, 3, 4, 5, 7, 9, 19, 25, 32, 47\} \]

According to those experimental data the percentage of primes is:

\[ \frac{10}{24} \approx 41.7\% \]

We considered 24 instead of 50 because we have excluded all the repeating terms in the sequence \( a(n) \) as already mentioned before. Based on that result the following conjecture can be formulated:

**Conjecture:** The number of primes generated by terms of Smarandache LCM sequence plus 1 is infinite.

Problem 2.

By using a Ubasic program we have found:

\[ \lim_{n \to \infty} \sum_{n} \frac{a(n)}{n!} \approx \sum_{n=1}^{\infty} \frac{1}{32 \cdot n^2 + 20 \cdot n - 11} = 4.195953... \]

Problem 3.

Always thanks to a Ubasic program the convergence value has been evaluated:

\[ \lim_{n \to \infty} \sum_{n} \frac{1}{a(n)} \approx \ln \frac{27773}{27281} = 1.7873... \]
where 27773 and 27281 are both prime numbers.

References.