PROPOSED PROBLEM (3)

Let \( \eta(n) \) be Smarandache Function: the smallest integer \( m \) such that \( m! \) is divisible by \( n \). Calculate \( \eta(p^{p+1}) \), where \( p \) is an odd prime number.

Solution.

The answer is \( p^2 \), because:

\[ p^2! = 1 \cdot 2 \cdot \ldots \cdot p \cdot \ldots \cdot (2p) \cdot \ldots \cdot ((p-1)p) \cdot \ldots \cdot (pp), \]

which is divisible by \( p^{p+1} \).

Any another number less than \( p^2 \) will have the property that its factorial is divisible by \( p^k \), with \( k < p + 1 \), but not divisible by \( p^{p+1} \).