SMARANDACHE REVERSE AUTO CORRELATED SEQUENCES AND SOME FIBONACCI DERIVED SMARANDACHE SEQUENCES

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Let \( a_1, a_2, a_3, \ldots \) be a base sequence. We define a Smarandache Reverse Auto-correlated Sequence (SRACS) \( b_1, b_2, b_3, \ldots \) as follows:

\[
b_1 = a_2^1, \quad b_2 = 2a_1a_2, \quad b_3 = a_2^2 + 2a_1a_3, \text{ etc. by the following transformation}
\]

\[
b_n = \sum_{k=1}^{n} a_k \cdot a_{n-k+1}
\]

and such a transformation as Smarandache Reverse Auto Correlation Transformation (SRACT)

We consider a few base sequences.

(1) \( 1, 2, 3, 4, 5, \ldots \)

i.e. \( 1C_1, 2C_1, 3C_1, 4C_1, 5C_1, \ldots \)

The SRACS comes out to be

\( 1, 4, 10, 20, 35, \ldots \) which can be rewritten as

i.e. \( 3C_3, 4C_3, 5C_3, 6C_3, 7C_3, \ldots \) we can call it SRACS(1)

Taking this as the base sequence we get SRACS(2) as

\( 1, 7, 36, 120, 330, \ldots \) which can be rewritten as

i.e. \( 7C_7, 8C_7, 9C_7, 10C_7, 11C_7, \ldots \) Taking this as the base sequence we get SRACS(3) as

\( 1, 16, 136, 816, 3876, \ldots \)

i.e. \( 15C_{15}, 16C_{15}, 17C_{15}, 18C_{15}, 19C_{15}, \ldots \)

This suggests the possibility of the following:

conjecture-I

The sequence obtained by \( 'n' \) times Smarandache Reverse Auto Correlation Transformation (SRACT) of the set of natural numbers is given by the following:

SRACS(n)

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The sequence obtained by \( n \) times Smarandache Reverse Auto Correlation transformation (SRACT) of the set of Triangular numbers is given by

**SRACT\((n)\)**

\[ b^{-1}C_{b^{-1}}, bC_{b^{-1}}, b^{+1}C_{b^{-1}}, b^{+2}C_{b^{-1}}, b^{+3}C_{b^{-1}}, \ldots \text{ where } h = 2^{n+1}. \]

(2) Triangular number as the base sequence:

1, 3, 6, 10, 15, 

i.e., \( ^2C_2, ^3C_2, ^4C_2, ^5C_2, ^6C_2, \ldots \)

The SRACS comes out to be

1, 6, 21, 56, 126, \ldots  which can be rewritten as

i.e., \( ^5C_5, ^6C_5, ^7C_5, ^8C_5, ^9C_5, \ldots \) we can call it SRACS(1)

Taking this as the base sequence we get SRACS(2) as

1, 12, 78, 364, 1365, \ldots

i.e., \( ^{11}C_{11}, ^{12}C_{11}, ^{13}C_{11}, ^{14}C_{11}, ^{15}C_{11}, \ldots \)

Taking this as the base sequence we get SRACS(3) as

1, 24, 300, 2600, 17550, \ldots

i.e., \( ^{23}C_{23}, ^{24}C_{23}, ^{25}C_{23}, ^{26}C_{23}, ^{27}C_{23}, \ldots \)

This suggests the possibility of the following conjecture-II

**Conjecture-II**

The sequence obtained by \( n \) times Smarandache Reverse Auto Correlation transformation (SRACT) of the set of Triangular numbers is given by

**SRACT\((n)\)**

\[ b^{-1}C_{b^{-1}}, bC_{b^{-1}}, b^{+1}C_{b^{-1}}, b^{+2}C_{b^{-1}}, b^{+3}C_{b^{-1}}, \ldots \text{ where } h = 3.2^n. \]

This can be generalised to conjecture the following:

**Conjecture-III**

Given the base sequence as \( ^nC_n, ^{n+1}C_n, ^{n+2}C_n, ^{n+3}C_n, ^{n+4}C_n, \ldots \)

The SRACS\((n)\) is given by

\[ b^{-1}C_{b^{-1}}, bC_{b^{-1}}, b^{+1}C_{b^{-1}}, b^{+2}C_{b^{-1}}, b^{+3}C_{b^{-1}}, \ldots \text{ where } h = (n+1).2^n. \]
SOME FIBONACCI DERIVED SMARANDACHE SEQUENCES

1. Smarandache Fibonacci Binary Sequence (SFBS):

In Fibonacci Rabbit problem we start with an immature pair 'I' which matures after one season to 'M'. This mature pair after one season stays alive and breeds a new immature pair and we get the following sequence

I → M → MI → MIM → MIMIM → MIMIMIM → MIMIMIMIMM

If we replace I by 0 and M by 1 we get the following binary sequence

0 → 1 → 10 → 101 → 10110 → 10110101 → 1011010110110

The decimal equivalent of the above sequences is

0 → 1 → 2 → 5 → 22 → 181 → 5814

we define the above sequence as the SFBS

We derive a reduction formula for the general term:

From the binary pattern we observe that

\[ T_n = T_{n-1} T_{n-2} \] (the digits of the \( T_{n-2} \) placed to the left of the digits of \( T_{n-1} \).)

Also the number of digits in \( T_r \) is nothing but the \( r \)-th Fibonacci number by definition. Hence we have

\[ T_n = T_{n-1} \cdot 2^{p(n-2)} + T_{n-2} \]

Problem: 1. How many of the above sequence are primes?

2. How many of them are Fibonacci numbers?

(2) Smarandache Fibonacci product Sequence:

The Fibonacci sequence is 1, 1, 2, 3, 5, 8, ... 

Take \( T_1 = 2 \), and \( T_2 = 3 \) and then \( T_n = T_{n-1} \cdot T_{n-2} \) we get the following sequence

2, 3, 6, 18, 108, 1944, 209952 -----(A)

In the above sequence which is just obtained by the first two terms, the whole Fibonacci sequence is inherent. This will be clear if we rewrite the above sequence as below:

\[ 2^1, 3^1, 2^1 3^1, 2^1 3^2, 2^2 3^2, 2^3 3^3, 2^5 3^5, \ldots \]

we have \( T_n = 2^{p(n-1)} \cdot 3^{p(n)} \)

The above idea can be extended by choosing \( r \) terms instead of two only and define

\( T_n = T_{n-1} T_{n-2} T_{n-3} \ldots T_{n-r} \) for \( n > r \).
Conjecture: (1) The following sequence obtained by incrementing the sequence (A) by 1
3, 4, 7, 19, 1945, 209953 \ldots contains infinitely many primes.

(2) It does not contain any Fibonacci number.