SMARANDACHE CONCATENATED POWER DECIMALS
AND
THEIR IRRATIONALITY

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Abstract

In this paper we prove that all Smarandache concatenated k-power decimals are irrational numbers.

For any positive integer k, we define the Smarandache concatenated k-power decimal \( \alpha_k \) as follows:

\[
\alpha_1 = 0.1234567891011..., \quad \alpha_2 = 0.149162536496481100121...
\]

(1)
\[
\alpha_3 = 0.18276412521634351272910001331..., ..., etc.
\]

In this paper we discuss the irrationality of \( \alpha_k \). We prove the following result:

Theorem. For any positive integer k, \( \alpha_k \) is an irrational number.

Proof. We now suppose that \( \alpha_k \) is a rational number.

Then, by \[1,\text{Theorem 135}\], \( \alpha_k \) is an infinite periodical decimal such that

\[
\alpha_k = 0.a_{r-1}a_{r-2}...a_1a_0.
\]

(2)

were \( r, t \) are fixed integers, with \( r \geq 0 \) and \( t > 0 \), \( a_1, ..., a_n, a_{r-t}, ..., a_{r-1} \) are integers satisfying

\( 0 \leq a_i \leq 9 \) (\( i = 1, 2, ..., r-t \)).

However, we see from (1) that there exist arbitrary many continuous zeros in the expansion of \( \alpha_k \). Therefore, we find from (2) that \( a_{r-t} = ... = a_{r-t} = 0 \). It implies that \( \alpha_k \) is a finite decimal; a contradiction.

Thus, \( \alpha_k \) must be an irrational number. The theorem is proved.

Finally, we pose a further question as follows:

Question. Is \( \alpha_k \) a transcendental number for any positive integer k?

By an old result of Mahler [2], the answer of our question is positive for k=1.

References: