SMARANDACHE FRIENDLY NUMBERS AND A FEW MORE SEQUENCES

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If the sum of any set of consecutive terms of a sequence = the product of the first and the last number of the set then this pair is called a Smarandache Friendly Pair with respect to the sequence.

{1} SMARANDACHE FRIENDLY NATURAL NUMBER PAIRS:

e.g. Consider the natural number sequence
1, 2, 3, 4, 5, 6, 7, ...
then the Smarandache friendly pairs are
(1,1), (3, 6), (15,35), (85, 204), ... etc.
as 3 + 4 + 5 +6 = 18 = 3 x 6
15 + 16 + 17 +...+ 33 + 34 + 35 = 525 = 15 x 35 etc.

There exist infinitely many such pairs. This is evident from the fact that if \((m, n)\) is a friendly pair then so is the pair \((2n+m, 5n +2m -1 )\). Ref [1].

{2} SMARANDACHE FRIENDLY PRIME PAIRS:

Consider the prime number sequence
2, 3, 5, 7, 11, 13, 17,23, 29, ...
we have 2 +3 + 5 = 10 = 2 x 5 , Hence (2, 5) is a friendly prime pair.
3 + 5 + 7 + 11 + 13 = 39 = 3 x 13 , (3,13) is a friendly prime pair.
5 + 7 + 11 +...+ 23 + 29 + 31 = 155 = 5 x 31 , ( 5, 31) is a friendly prime pair.

Similarly ( 7, 53 ) is also a Smarandache friendly prime pair. In a friendly prime pair \((p, q)\) we define \(q\) as the big brother of \(p\).

Open Problems: (1) Are there infinitely many friendly prime pairs?

2. Are there big brothers for every prime?

{3} SMARANDACHE UNDER-FRIENDLY PAIR:

If the sum of any set of consecutive terms of a sequence is a divisor of the product of the first and the last number of the set then this pair is called a Smarandache under- Friendly Pair with respect to the sequence.
{4} SMARANDACHE OVER-FRIENDLY PAIR:

If the sum of any set of consecutive terms of a sequence is a multiple of the product of the first and the last number of the set then this pair is called a Smarandache Over-Friendly Pair with respect to the sequence.

{5} SMARANDACHE SIGMA DIVISOR PRIME SEQUENCE:

The sequence of primes \( p_n \), which satisfy the following congruence.

\[
\sum_{r=1}^{n-1} p_r \equiv 0 \pmod{p_n}
\]

2, 5, 71, ...

5 divides 10, and 71 divides 568 = 2 + 3 + 5 + \ldots + 67

Problems: (1) Is the above sequence infinite?

Conjecture: Every prime divides at least one such cumulative sum.

{6} SMARANDACHE SMALLEST NUMBER WITH \( 'n' \) DIVISORS SEQUENCE:

1, 2, 4, 6, 10, 12, 24, 36, 48, 1024, ...

d(1) = 1, d(2) = 2, d(4) = 3, d(6) = 4, d(10) = 5, d(12) = 6 etc., d(1024) = n, where \( T_n \) is smallest such number.

It is evident \( T_p = 2^{p-1} \), if \( p \) is a prime.

The sequence \( T_n+1 \) is

2, 3, 5, 7, 17, 13, 65, 25, 37, 49, 1025, ...

Conjectures: (1) The above sequence contains infinitely many primes.

(2) The only Mersenne's prime it contains is 7.

(3) The above sequence contains infinitely many perfect squares.

{7} SMARANDACHE INTEGER PART \( k^* \) SEQUENCE ( SIPS):

**In this sequence \( k \) is a non integer. For example:

(i) SMARANDACHE INTEGER PART \( \pi^* \) SEQUENCE:

\[ [\pi^1], [\pi^2], [\pi^3], [\pi^4], \ldots \]

3, 9, 31, 97, ...
(ii) SMARANDACHE INTEGER PART e^r SEQUENCE:
\[ e^1, [e^2], [e^3], [e^4], \ldots \]
2, 7, 20, 54, 148, 403, \ldots

Conjecture: Every SIPS contains infinitely many primes.

{8} Smarandache Summable Divisor Pairs (SSDP):
Pair of numbers (m,n) which satisfy the following relation
\[ d(m) + d(n) = d(m + n) \]
e.g. we have \( d(2) + d(10) = d(12) \), \( d(3) + d(5) = d(8) \), \( d(4) + d(256) = d(260) \),
\[ d(8) + d(22) = d(30) \text{, etc.} \]
hence (2, 10), (3, 5), (4, 256), (8, 22) are SSDPs.

Conjecture: (1) There are infinitely many SSDPs?
(2) For every integer m there exists a number n such that (m,n) is an SSDP.

{9} SMARANDACHE REIMANN ZETA SEQUENCE
6, 90, 945, 93555, 638512875, \ldots

where \( T_n \) is given by the following relation of
\[ z(s) = \sum \frac{n^s}{\pi^{2s} / T_n} \]

Conjecture: No two terms of this sequence are relatively prime.

Consider the sequence obtained by incrementing each term by one
7, 91, 946, 9451, 93556, 638512876, \ldots

Problem: How many primes does the above sequence contain?

{10} SMARANDACHE PRODUCT OF DIGITS SEQUENCE:
The \( n \)th term of this sequence is defined as \( T_n = \text{product of the digits of } n \).
1, 2, 3, 4, 5, 6, 7, 8, 9, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, 2, 4, 6, 8, 10, 12, \ldots

{11} SMARANDACHE SIGMA PRODUCT OF DIGITS NATURAL SEQUENCE:
The $n^{th}$ term of this sequence is defined as the sum of the products of all the numbers from 1 to $n$.

$1, 3, 6, 10, 15, 21, 28, 36, 45, 45, 46, 48, 51, 55, 60, 66, 73, 81, 90, 90, 92, 96, \ldots$

Here we consider the terms of the sequence for some values of $n$.
For $n = 9$ we have $T_n = 45$, The sum of all the single digit numbers = 45
For $n = 99$ we have $T_n = 2070 = 45^2 + 45$.
Similarly we have $T_{999} = (T_9)^3 + (T_9)^2 + T_9 = 45^3 + 45^2 + 45 = (45^4 - 1) / (45 - 1) = (45^4 - 1) / 44$
The above proposition can easily be proved.
This can be further generalized for a number system with base 'b' ($b = 10$, the decimal system has already been considered.)

For a number system with base 'b' the $(b^r - 1)^{th}$ term in the Smarandache sigma product of digits sequence is

$$2\left\lfloor \frac{b(b-1)/2}{b^r - 1} \right\rfloor / \{ b^2 - b - 2 \}$$

Further Scope: The task ahead is to find the $n^{th}$ term in the above sequence for an arbitrary value of $n$.

{12} SMARANDACHE SIGMA PRODUCT OF DIGITS ODD SEQUENCE:
1, 4, 9, 16, 25, 26, 29, 34, 41, 50, 52, 58, 68, 82, 100, 103, 112, 127, 148, \ldots

It can be proved that for $n = 10^r - 1$, $T_n$ is the sum of the $r$ terms of the Geometric progression with the first term as 25 and the common ratio as 45.

{13} SMARANDACHE SIGMA PRODUCT OF DIGITS EVEN SEQUENCE:
2, 6, 12, 20, 20, 22, 26, 32, 40, 40, 44, 52, 62, 78, 78, 84, 96, 114, 138, \ldots

It can again be proved that for $n = 10^r - 1$, $T_n$ is the sum of the $r$ terms of the Geometric progression with the first term as 20 and the common ratio as 45.

Open Problem: Are there infinitely many common members in {12} and {13}?

Reference: