SMARANDACHE PARTITION TYPE AND OTHER SEQUENCES*

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ABSTRACT

Thanks to C. Dumitrescu and Dr. V. Seleacu of the University of Craiova, Department of Mathematics, I became familiar with some of the Smarandache Sequences. I list some of them, as well as questions related to them. Now I'm working in a few conjectures involving these sequences.

Examples of Smarandache Partition type sequences:

A. 1, 1, 1, 2, 2, 2, 3, 4, 4, ....

(How many times is n written as a sum of non-null squares, disregarding the order of the terms:
for example:

\[ 9 = 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2 \]
\[ = 1^2 + 1^2 + 1^2 + 1^2 + 2^2 \]
\[ = 1^2 + 2^2 + 2^2 \]
\[ = 3^2, \]

therefore \( ns(9) = 4. \)

B. 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 5, 5, 5, 6, 6, ...

(How many times is n written as a sum of non-null cubes, disregarding the order of the terms:
for example:

\[ 9 = 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3 \]
\[ = 1^3 + 2^3, \]

therefore, \( nc(9) = 2. \)

C. General-partition type sequence:

Let \( f \) be an arithmetic function and \( R \) a relation among numbers.
(How many times can \( n \) be written under the form:

\[ n = R(f(n_1), f(n_2), \ldots, f(n_k)) \]

for some \( k \) and \( n_1, n_2, \ldots, n_k \) such that

\[ n_1 + n_2 + \ldots + n_k = n? \]
Examples of other sequences:

(1) Smarandache Anti-symmetric sequence:

11, 1212, 123123, 12341234, 1234512345, 123456123456,
12345671234567, 1234567812345678, 123456789, 123456789,
1234567891012345678910, 1234567891011, 1234567891011, ...

(2) Smarandache Triangular base:

1, 2, 10, 11, 12, 100, 101, 102, 110, 1000, 1001, 1002, 1010, 1011,
10000, 10001, 10002, 10010, 10011, 10012, 100000, 100001, 100002,
100010, 100011, 100012, 100100, 1000000, 1000001, 1000002, 1000010,
1000011, 1000012, 1000100, ...

(Numbers written in the triangular base, defined as follows:

\[ t(n) = \frac{(n(n+1))}{2}, \text{ for } n \geq 1. \]

(3) Smarandache Double factorial base:

1, 10, 100, 101, 110, 200, 201, 1000, 1001, 1010, 1100, 1101, 1110,
1200, 10000, 10001, 10010, 10100, 10101, 10110, 10200, 10201, 11000,
11001, 11010, 11100, 11101, 11110, 11200, 11201, 12000, ...

(Numbers written in the double factorial base, defined as follows:

\[ df(n) = n!! \]

(4) Smarandache Non-multiplicative sequence:

General definition: Let \( m_1, m_2, \ldots, m_k \) be the first \( k \) terms of the sequence, where \( k \geq 2 \);

then \( m_i \), for \( i \geq k+1 \), is the smallest number not equal to the product of \( k \) previous distinct terms.

(5) Smarandache Non-arithmetic progression:

1, 2, 4, 5, 10, 11, 13, 14, 28, 29, 31, 32, 37, 38, 40, 41, 64, ...

General definition: if \( m_1, m_2 \), are the first two terms of the sequence,

then \( m_k \), for \( k \geq 3 \), is the smallest number such that no 3-term arithmetic
progression is in the sequence.

In our case the first two terms are 1, respectively 2.

Generalization: same initial conditions, but no i-term arithmetic progression
in the sequence (for a given \( i \geq 3 \)).
(6) Smarandache Prime product sequence:

\[ P_n = 1 + p_1 p_2 \ldots p_n, \text{ where } p_k \text{ is the } k\text{-th prime.} \]

Question: How many of them are prime?

(7) Smarandache Square product sequence:

\[ S_n = 1 + s_1 s_2 \ldots s_n, \text{ where } s_k \text{ is the } k\text{-th square number.} \]

Question: How many of them are prime?

(8) Smarandache Cubic product sequence:

\[ C_k = 1 + c_1 c_2 \ldots c_k, \text{ where } c_k \text{ is the } k\text{-th cubic number.} \]

Question: How many of them are prime?

(9) Smarandache Factorial product sequence:

\[ F_k = 1 + f_1 f_2 \ldots f_k, \text{ where } f_k \text{ is the } k\text{-th factorial number.} \]

Question: How many of them are prime?

(10) Smarandache U-product sequence (generalization):

Let \( u_n, n \geq 1 \) be a positive integer sequence. Then we define a U-sequence as follows:

\[ U_n = 1 + u_1 u_2 \ldots u_n. \]

(11) Smarandache Non-geometric progression.

\[ 1, 2, 3, 5, 6, 7, 8, 10, 11, 13, 14, 15, 16, 17, 19, 21, 22, 23, 24, 26, 27, 29, 30, 31, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 45, 47, 48, 50, 51, 53, \ldots \]

General definition: if \( m_1, m_2 \), are the first two terms of the sequence, then \( m_k \), for \( k \geq 3 \), is the smallest number such that no 3-term geometric progression is in the sequence. In our case the first two terms are 1, respectively 2.
(12) Smarandache Unary sequence:

\[ u(n) = 11...1, \ p_n \ \text{digits of "1"}, \ \text{where} \ p_n \ \text{is the n-th prime.} \]

The old question: are there are infinite number of primes belonging to the sequence?

(13) Smarandache No-prime-digit sequence:

\[ 1, 4, 6, 8, 9, 10, 1 1, 1, 14, 1, 16, 1, 18, 19, 0, 1, 4, 6, 8, 9, 0 1, 4, 6, 8, 9, 40, 41, 42, 4, 44, 4, 46, 48, 49, 0, \ldots \]

(Take out all prime digits of \( n \).)

(14) Smarandache No-square-digit-sequence.

\[ 2, 3, 5, 6, 7, 8, 2, 3, 5, 6, 7, 8, 2, 22, 23, 2, 25, 26, 27, 28, 2, 3, 32, 33, 3, 35, 36, 37, 38, 3, 2, 3, 5, 6, 7, 8, 5, 5, 52, 53, 5, 55, 56, 57, 58, 5, 6, 6, 62, \ldots \]

(Take out all square digits of \( n \).)