Consider a rectangular city with a mesh of tracks which are of equal length and which are either horizontal or vertical and meeting at nodes. If one row contains $m$ tracks and one column contains $n$ tracks then there are $(m+1)(n+1)$ nodes. To begin with let the city be of a square shape i.e. $m = n$.

Consider the possible number of routes $R$ which a person at one end of the city can take from a source $S$ (starting point) to reach the diagonally opposite end $D$ the destination.

For $m = 1$ Number of routes $R = 1$
For $m = 2$, $R = 2$
For $m = 3$, $R = 12$

We see that for the shortest routes one has to travel $2m$ units of track length. There are routes with $2m + 2$ units up to the longest route being $4m + 4$.

We define **Smarandache Route Sequence (SRS)** as the number of all possible routes for a $m$ square city. This includes routes with path lengths ranging from $2m$ to $4m + 4$.

**Open problem(1): To derive a reduction formula/ general formula for SRS.**

Here we derive a reduction formula, thus a general formula for the number of shortest routes.

**Reduction formula for number of shortest routes:**

Refer figure -II

Let $R_{jk}$ = number of routes to reach node $(j, k)$.  

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Node \((j, k)\). Can be reached only either from node \((j-1, k)\) or from the node \((j, k-1)\). *{As only shortest routes are to be considered}.

It is clear that there is only one way of reaching node \((j, k)\) from node \((j-1, k)\). Similarly there is only one way of reaching node \((j, k)\) from node \((j, k-1)\). Hence the number of shortest routes to node \((j, k)\) is given by

\[ R_{j,k} = R_{j-1,k} + 1 \cdot R_{j,k-1} = R_{j-1,k} + R_{j,k-1} \]

This gives the reduction formula for \(R_{j,k}\).

Applying this reduction formula to fill the chart we observe that the total number of shortest routes to the destination (the other end of the diagonal) is \(2^k \cdot C_n\). This can be established by induction.

We can further categorize the routes by the number of turning points it is subjected to.

The chart for various number of turning points (TPs) for a city with 9 nodes is given below.

<table>
<thead>
<tr>
<th>No of TPs</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of routes</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

Further Scope:

(1) To explore for patterns among total number of routes, number of turning points and develop formulae for square as well as rectangular meshes (cities).

(2) To study as to how many routes pass through a given number/set of nodes? How many of them pass through all the nodes?