SOME NOTIONS ON LEAST COMMON MULTIPLES

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In [1] Smarandache LCM Sequence has been defined as \( T_n = \text{LCM (1 to } n) \) = LCM of all the natural numbers up to \( n \).

The SLS is

\[
1, 2, 6, 60, 840, 2520, 2520, \ldots
\]

We denote the LCM of a set of numbers \( a, b, c, \) etc. as \( \text{LCM}(a, b, c, \) etc.\)

We have the well known result that \( n! \) divides the product of any set of \( n \) consecutive numbers. Using this idea we define Smarandache LCM Ratio Sequence of the \( r^\text{th} \) kind as SLRS(\( r \))

The \( n^\text{th} \) term \( T_n = \text{LCM (n, n+1, n+2, \ldots n+r-1)}/\text{LCM (1, 2, 3, 4, \ldots r)} \)

As per our definition we get SLRS(1) as

\[
1, 5, 5, 35, 70, 42, 210, \ldots
\]

It can be noticed that for \( r > 2 \) the terms do not follow any visible patterns.

OPEN PROBLEM: To explore for patterns/ find reduction formulae for \( T_n \).

Definition: Like \(^4C_r\), the combination of \( r \) out of \( n \) given objects, We define a new term \(^*L_r\)

As

\[
(^*L_r = \text{LCM (n, n-1, n-2, \ldots n-r+1)}/\text{LCM (1, 2, 3, \ldots r)}
\]

(Numerator is the LCM of \( n, n-1, n-2, \ldots n-r+1 \) and the denominator is the LCM of first natural numbers.)

we get \(^1L_0 = 1, ^1L_1 = 1, ^2L_0 = 1, ^2L_1 = 2, ^2L_2 = 2 \) etc. define \(^6L_0 = 1\)

we get the following triangle:

\[
1, 1 \\
1, 2, 1 \\
1, 3, 3, 1 \\
1, 4, 6, 2, 1 \\
1, 5, 10, 10, 5, 1
\]

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1, 6, 15, 10, 5, 1, 1
1, 7, 21, 35, 35, 7, 1
1, 8, 28, 28, 70, 14, 14, 2, 1
1.9, 36, 84, 42, 42, 42, 6, 3, 1
1, 10, 45, 60, 210, 42, 42, 6, 3, 1

Let this triangle be called **Smarandache AMAR LCM Triangle**

**Note:** As \( r! \) divides the product of \( r \) consecutive integers so does the LCM \( \{1, 2, 3, \ldots, r\} \) divide the LCM of any \( r \) consecutive numbers. Hence we get only integers as the members of the above triangle.

Following properties of **Smarandache AMAR LCM Triangle** are noticable.

1. The first column and the leading diagonal elements are all unity.
2. The \( k^{th} \) column is nothing but the SLRS\((k)\).
3. The first four rows are the same as that of the Pascal's Triangle.
4. \( \text{II}^{th} \) column contains natural numbers.
5. \( \text{III}^{rd} \) column elements are the triangular numbers.
6. If \( p \) is a prime then \( p \) divides all the terms of the \( p^{th} \) row except the first and the last which are unity. In other words \( \sum p^{th} \text{ row} = 2 (\mod {p}) \)

Some keen observation opens up vistas of challenging problems:
In the \( 9^{th} \) row 42 appears at three consecutive places.

**OPEN PROBLEM:**
1. Can there be arbitrarily large lengths of equal values appear in a row?
2. To find the sum of a row.
3. Explore for congruence properties for composite \( n \).

**SMARANDACHE LCM FUNCTION:**
The Smarandache function \( S(n) \) is defined as \( S(n) = k \) where is the smallest integer such that \( n \) divides \( k! \). Here we define another function as follows:

**Smarandache Lcm Function**
\[ S_L(n) = k \]
where \( k \) is the smallest integer such that \( n \) divide LCM \( \{1, 2, 3 \ldots k\} \).
Let \( n = p_1^{a_1} p_2^{a_2} p_3^{a_3} \ldots p_r^{a_r} \)
Let \( p_m \) be the largest divisor of \( n \) with only one prime factor, then
We have \( S_L(n) = p_m^{a_m} \)
If \( n = k! \) then \( S(n) = k \) and \( S_L(n) > k \)
If \( n \) is a prime then we have \( S_L(n) = S(n) = n \)
Clearly \( S_L(n) \geq S(n) \) the equality holding good for \( n \) a prime or \( n = 4, n=12 \).
Also \( S_L(n) = n \) if \( n \) is a prime power. \( (n = p^r) \)

**OPEN PROBLEMS:**
1. Are there numbers \( n > 12 \) for which \( S_L(n) = S(n) \).
2. Are there numbers \( n \) for which \( S_L(n) = S(n) \neq n \)

**REFERENCE:**