THE 2-DIVISIBILITY OF EVEN ELEMENTS OF THE
SMARANDACHE DECONSTRUCTIVE SEQUENCE

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Abstract. In this paper we prove that if \( n > 5 \) and \( SDS(n) \) is even, then \( SDS(n) \) is exactly divisible by \( 2^7 \).

Key words. Smarandache deconstructive sequence, 2-divisibility.

The Smarandache deconstructive sequence is constructed by sequentially repeating the digits 1, 2, \( \ldots \), 9 in the following way:

\[
1, 23, 456, 7891, \ldots
\]

which first appeared in [3]. For any positive integer \( n \), let \( SDS(n) \) denote the \( n \)-th element of the Smarandache deconstructive sequence. In [1], Ashbacher considered the values of the first thirty elements of this sequence. He showed that \( SDS(3) = 456 \) is divisible by \( 2^3 \), \( SDS(5) = 23456 \) by \( 2^5 \) and all others by \( 2^7 \). Therefore, Ashbacher proposed the following question.

Question. If we form a sequence from the elements \( SDS(n) \) which the trailing digits are 6, do the powers of 2 that divide them form a monotonically increasing sequence?

In this paper we completely solve the mentioned question. We prove the following result.

Theorem. If \( n > 5 \) and \( SDS(n) \) is even, then \( SDS(n) \) is exactly divisible by \( 2^7 \).

Proof. By the result of [2], if \( SDS(n) \) is even, then the trailing digit of it must be 6. Moreover, if \( n > 5 \),
then \( n \geq 12 \). Therefore, by (1), if \( n > 5 \) and \( SDS(n) \) is even, then we have

\[
S(n) = 89123456 + k \cdot 10^8,
\]

where \( k \) is a positive integer. Notice that \( 2^8 \mid 10^8 \) and \( 2^7 \mid 89123456 \). We see from (2) that \( 2^7 \mid SDS(n) \). Thus, the theorem is proved.

References


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