THE DIVISIBILITY OF THE SMARANDACHE
COMBINATORIAL SEQUENCE OF DEGREE TWO

Maohua Le
Department of Mathematics
Zhanjiang Normal College
29 Cunjin Road, Chikan
Zhanjiang, Guangdong
P.R.China

Abstract: In this paper we prove that there has only the consecutive
terms of the Smarandache combinatorial sequence of degree two are
pairwise coprime.

Key words: Smarandache combinatorial sequences; consecutive
terms; divisibility

Let \( r \) be a positive integer with \( r > 1 \). Let \( SCS(r) = \{a(r,n)\}_{n=1}^{\infty} \) be
the Smarandache combinatorial sequence of degree \( r \). Then we have
\( a(r,n) = n(n=1,2,\ldots,r) \) and \( a(r,n)(n > r) \) is the sum of all the products
of the previous terms of the sequence taking \( r \) terms at a time. In [2],
Murthy asked that how many of the consecutive terms of \( SCS(r) \) are
pairwise coprime.

In this paper we solve this problem for \( r = 2 \). We prove the
following result.

Supported by the National Natural Science Foundation of China
(No.10271104), the Guangdong Provincial Natural Science Foundation
(No.011781) and the Natural Science Foundation of the Education
Department of Guangdong Province (No.0161).
Theorem. For any positive integer \( n \), we have \( a(2, n+1) \equiv 0 \) (mod \( a(2,n) \)).

By the above mentioned theorem, we obtain the following corollary immediately.

Corollary. There has only the consecutive terms 1,2 of \( SCS(2) \) are pairwise coprime.

Proof of Theorem. Let \( b(n) = a(2,n) \) for any \( n \). Then we have \( b(1) = 1 \) and \( b(2) = 2 \). It implies that the theorem holds for \( n=1 \).

By the define of \( SCS(2) \), if \( n > 1 \), then we have

\[
b(n) = b(1)b(2) + \cdots + b(n-2)b(n-1)
\]

\[
= \frac{1}{2} \left( (b(1) + \cdots + b(n-1))^2 - \left( b^2(1) + \cdots + b^2(n-1) \right) \right)
\]

and

\[
b(n) = b(1)b(2) + \cdots + b(n-2)b(n-1)
\]

\[
= \frac{1}{2} \left( (b(1) + \cdots + b(n-1) + b(n))^2 - \left( b^2(1) + \cdots + b^2(n-1) + b^2(n) \right) \right)
\]

using the basic properties of congruence (see [1, Chapter V]), we get from (1) and (2) that

\[
b(n+1) = \frac{1}{2} \left( (b(1) + \cdots + b(n-1))^2 - \left( b^2(1) + \cdots + b^2(n-1) \right) \right)
\]

\[
\equiv b(n) \equiv 0 \text{ (mod } b(n) \text{)}.
\]

Thus, the theorem is proved.

References
