THE POWERS IN THE SMARANDACHE CUBIC PRODUCT SEQUENCES

Maohua Le

Abstract. Let $P$ and $Q$ denote the Smarandache cubic product sequences of the first kind and the second kind respectively. In this paper we prove that $P$ contains only one power $9$ and $Q$ does not contain any power.

Key words. Smarandache cubic product sequence, power.

For any positive integer $n$, Let $C(n)$ be the $n$-th cubic. Further, let

(1) $P(n) = \prod_{k=1}^{n} C(k) + 1$

and

(2) $Q(n) = \prod_{k=1}^{n} C(k) - 1$.

Then the sequences $P = \{P(n)\}_{n=1}^{\infty}$ and $Q = \{Q(n)\}_{n=1}^{\infty}$ are called the Smarandache cubic product sequence of the first kind and the second kind respectively (see [5]). In this paper we consider the powers in $P$ and $Q$. We prove the following result.

Theorem. The sequence $P$ contains only one power $P(2) = 3^2$. The sequence $Q$ does not contain any power.

Proof. If $P(n)$ is a power, then from (1) we get

(3) $(n!)^3 + 1 = a^r$,
where $a$ and $r$ are positive integers satisfying $a>1$ and $r>1$. By (3), if $2 \mid r$, then the equation
\[(n!)^2 + 1 = a^r,\]
has a positive integer solution $(X,Y) = (n!, a^2r)$. Using a well known result of Euler (see [3,p.302]), (4) has only one positive integer solution $(X,Y) = (2,3)$. It implies that $P$ contain only one power $P(2) = 3^2$ with $2 \mid r$. If $2 \nmid r$, then the equation
\[X^3 + 1 = Y^2,\]
has a positive integer solution $(X,Y) = (n!, a^2r)$. However, by [4], it is impossible. Thus, $P$ contains only one power $P(2) = 3^2$.

Similarly, by (2), if $Q(n)$ is a power, then we have
\[(n!)^3 - 1 = a^r,\]
where $a$ and $r$ are positive integers satisfying $a>1$ and $r>1$. It implies that the equation
\[X^3 - 1 = Y^m, m > 1,\]
has a positive integer solution $(X,Y,m) = (n!, a^r)$. However, by the results of [2] and [4], it is impossible. Thus, the sequence $Q$ does not contain any power. The theorem is proved.

References


Department of Mathematics
Zhanjiang Normal College
Zhanjiang, Guangdong
P.R. CHINA