THE PRIMES IN THE SMARANDACHE POWER PRODUCT SEQUENCES OF THE SECOND KIND

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Abstract. In this paper we completely determine the primes in the Smarandache power product sequences of the second kind.

Key words. Smarandache power product sequences, second kind, prime.

For any positive integers $n, r$ with $r > 1$, let $P(n, r)$ be the $n$-th power of degree $r$. Further, let

\[ U(n, r) = \prod_{k=1}^{n} P(k, r) - 1. \]

Then the sequence $U(r) = \{ U(n, r) \}_{n=1}^{\infty}$ is called the Smarandache $r$-power product sequence of the second kind. In [2], Russo proposed the following question.

Question. How many terms in $U(2)$ and $U(3)$ are primes?

In this paper we completely solve the mentioned question. We prove a more strong result as follows.

Theorem. If $r$ and $2^r - 1$ are both primes, then $U(r)$ contains only one prime $U(2, r) = 2^r - 1$. Otherwise, $U(r)$ does not contain any prime.

Proof. Since $U(1, r) = 0$, we may assume that $n > 1$. By (1), we get

\[ U(n, r) = (n!)^r - 1 = (n! - 1)((n!)^{r-1} + (n!)^{r-2} + \cdots + 1). \]

Since $n! > 2$ if $n > 2$, we see from (2) that $U(n, r)$ is not a prime if $n > 2$. When $n = 2$, we get from (2) that
(3) \[ U(2,r) = 2^r - 1. \]
Therefore, by [1, Theorem 18], we find from (3) that \( U(r) \) contains a prime if and only if \( r \) and \( 2^r - 1 \) are both primes. The theorem is proved.

References


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