THE SQUARES IN THE SMARANDACHE HIGHER POWER PRODUCT SEQUENCES

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Abstract. In this paper we prove that the Smarandache higher power product sequences of the first kind and the second kind do not contain squares.

Key words. Smarandache product sequence, higher power, square.

Let \( r \) be a positive integer with \( r > 3 \), and let \( A(n) \) be the \( n \)-th power of degree \( r \). Further, let

\[
\begin{align*}
\text{(1)} & \\
p(n) &= \prod_{k=1}^{n} A(k) + 1
\end{align*}
\]

and

\[
\begin{align*}
\text{(2)} & \\
Q(n) &= \prod_{k=1}^{n} A(k) - 1.
\end{align*}
\]

Then the sequences \( P = \{P(n)\}_{n=1}^{\infty} \) and \( Q = \{Q(n)\}_{n=1}^{\infty} \) are called the Smarandache higher power product sequences of the first kind and the second kind respectively. In this paper we consider the squares in \( P \) and \( Q \). We prove the following result.

Theorem. For any positive integer \( r \) with \( r > 3 \), the sequences \( P \) and \( Q \) do not contain squares.

Proof. By (1), if \( P(n) \) is a square, then we have

\[
\begin{align*}
\text{(3)} & \\
(n!)^r + 1 &= a^2,
\end{align*}
\]

where \( a \) is a positive integer. It implies that the equation

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(4) \[ x^m+1=y^2, \quad m>3 \]
has a positive integer solution \((x,y,m)=(n!,a,r)\). However, by
the result of \([1]\), the equation (4) has no positive integer
solution \((x,y,m)\). Thus, the sequence \(P\) does not contain
squares.

Similarly, by (2), if \(Q(n)\) is a square, then we have
\[(5) \quad (n!)y-1=a^2,\]
where \(a\) is a positive integer. It implies that the
equation
\[(6) \quad x^m-1=y^2, m>3.\]
has a positive integer solution \((x,y,m)=(n!,a,r)\). However, by
the result of \([2]\), it is impossible. Thus, the sequence \(Q\)
does not contain squares. The theorem is proved.

References

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