Theorem: If $p$ is a real number $\geq 1$ and $a_i^{(k)} \in \mathbb{R}^+$ with $i \in \{1, 2, ..., n\}$ and $k \in \{1, 2, ..., m\}$, then:

$$\left( \sum_{i=1}^{n} \left( \sum_{k=1}^{m} a_i^{(k)} \right)^p \right)^{1/p} \leq \left( \sum_{k=1}^{m} \left( \sum_{i=1}^{n} a_i^{(k)} \right)^p \right)^{1/p}$$

Demonstration by recurrence on $m \in \mathbb{N}^*$.

First of all one shows that:

$$\left( \sum_{i=1}^{n} \left( a_i^{(1)} \right)^p \right)^{1/p} \leq \left( \sum_{i=1}^{n} \left( a_i^{(1)} \right)^p \right)^{1/p}$$

which is obvious, and proves that the inequality is true for $m = 1$.

(The case $m = 2$ precisely constitutes the inequality of Minkowski, which is naturally true!).

Let us suppose that the inequality is true for all the values less or equal to $m$

$$\left( \sum_{i=1}^{n} \left( \sum_{k=1}^{m+1} a_i^{(k)} \right)^p \right)^{1/p} \leq \left( \sum_{i=1}^{n} \left( a_i^{(1)} \right)^p \right)^{1/p} + \left( \sum_{k=2}^{m+1} \left( \sum_{i=1}^{n} a_i^{(k)} \right)^p \right)^{1/p}$$

$$\leq \left( \sum_{i=1}^{n} \left( a_i^{(1)} \right)^p \right)^{1/p} + \sum_{k=2}^{m+1} \left( \sum_{i=1}^{n} a_i^{(k)} \right)^p$$

and this last sum is $\left( \sum_{k=1}^{m+1} \left( \sum_{i=1}^{n} a_i^{(k)} \right) \right)^{1/p}$ therefore the inequality is true for the level $m + 1$. 