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Generalization of An Er’s Matrix Method for Computing

GENERALIZATION OF AN ER’S MATRIX METHOD FOR COMPUTING

Er’s matrix method for computing Fibonacci numbers and their sums can be extended to the s-additive sequence:

\[ g_{-s+1} = g_{-s+2} = \ldots = g_1 = 0, \quad g_0 = 1, \]

and

\[ g_n = \sum_{i=1}^{s} g_{n-i} \quad \text{for} \quad n > 0. \]

For example, if we note \( S_n = \sum_{j=1}^{n} g_j \), we define two \((s+1) \times (s+1)\) matrixes such that:

\[
B_n = \begin{bmatrix}
1 & 0 & 0 & \ldots & 0 & 0 \\
S_n & g_n & g_{n-1} & \ldots & g_{n-s+2} & g_{n-s+1} \\
: & : & : & \ldots & : & : \\
S_{n-s+1} & g_{n-s+1} & g_{n-s} & \ldots & g_{n-2s+3} & g_{n-2s+2} \\
\end{bmatrix},
\]

\( n \geq 1, \) and

\[
M = \begin{bmatrix}
1 & 0 & 0 & \ldots & 0 \\
1 & 1 & 1 & \ldots & 0 \\
: & : & : & \ldots & : \\
1 & 1 & 0 & \ldots & 1 \\
1 & 1 & 0 & \ldots & 0 \\
\end{bmatrix},
\]

thus, we have analogously:

\[ B_{n+1} = M^{n+1}, \quad M^{r \cdot c} = M^r \cdot M^c, \]

whence

\[
S_{r+c} = S_r + g_r S_c + g_{r-1} S_{c-1} + \ldots + g_{r-s+1} S_{c-s+1},
\]

\[
g_{r+c} = g_r g_c + g_{r-1} g_{c-1} + \ldots + g_{r-s+1} g_{c-s+1},
\]

and for \( r = c = n \) it results:

\[
S_{2n} = S_n + g_n S_n + g_{n-1} S_{n-1} + \ldots + g_{n-s+1} S_{n-s+1},
\]

\[
g_{2n} = g_n^2 + g_{n-1}^2 + \ldots + g_{n-s+1}^2,
\]

for \( r = n, \quad c = n - 1, \) we find:

\[
g_{2n-1} = g_n g_{n-1} + g_{n-2} g_{n-2} + \ldots + g_{n-s+1} g_{n-s}, \quad \text{etc.}
\]

\[
S_{2n-1} = S_n + g_n S_{n-1} + g_{n-1} S_{n-2} + \ldots + g_{n-s+1} S_{n-s}
\]
Whence we can construct a similar algorithm as M. C. Er for computing s-additive numbers and their sums.

REFERENCE:


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