A Group-Permutation Algorithm to Solve the Generalized SUDOKU

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Sudoku is a game with numbers, formed by a square with the side of 9, and on each row and column are placed the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, written only one time; the square is subdivided in 9 smaller squares with the side of 3 × 3, which, also, must satisfy the same condition, i.e. each square to contain all digits from 1 to 9 written only once.

The Japanese company Nikoli has popularized this game in 1986 under the name of sudoku, meaning “single number”.

Sudoku can be generalized to squares whose dimensions are \( n^2 \times n^2 \), where \( n \geq 2 \), using various symbols (numbers, letters, mathematical symbols, etc.), written just one time on each row and on each column; and the large square is divided into \( n^2 \) small squares with the side \( n \times n \) and each will contain all \( n^2 \) symbols written only once.

An elementary solution of one of these generalized Sudokus, with elements (symbols) from the set

\[ S = \{ s_1, s_2, \ldots, s_n, s_{n+1}, \ldots, s_{2n}, \ldots, s_{n^2} \} \]

(supposing that their placement represents the relation of total order on the set of elements \( S \))

is:

Row 1: all elements in ascending order

\[ s_1, s_2, \ldots, s_n, s_{n+1}, \ldots, s_{2n}, \ldots, s_{n^2} \]

On the next rows we will use circular permutations, considering groups of \( n \) elements from the first row as follows:

Row 2:

\[ s_{n+1}, s_{n+2}, \ldots, s_{2n}; s_{2n+1}, \ldots, s_{3n}; \ldots, s_{n^2}; s_{1}, s_{2}, \ldots, s_{n} \]

Row 3:

\[ s_{2n+1}, \ldots, s_{3n}; s_{n^2}; s_{1}, s_{2}, \ldots, s_{n}; s_{n+1}, s_{n+2}, \ldots, s_{2n} \]

Row \( n \):

\[ s_{n^2-n+1}, \ldots, s_{n^2}; s_{1}, \ldots, s_{n}, s_{n+1}, s_{n+2}, \ldots, s_{2n}; \ldots, s_{3n}; \ldots, s_{n^2-n} \]

Now we start permutations of the elements of row \( n+1 \) considering again groups of \( n \) elements.

Row \( n+1 \):

\[ s_{2}, \ldots, s_{n}, s_{n+1}; s_{n+2}, \ldots, s_{2n}; s_{2n+1}, \ldots, s_{n^2-n+2}, \ldots, s_{n^2}, s_{1} \]

Row \( n+2 \):

\[ s_{n+2}, \ldots, s_{2n}, s_{2n+1}, \ldots, s_{n^2-n+2}, \ldots, s_{n^2}, s_{1}, s_{2}, \ldots, s_{n}, s_{n+1} \]
Row $2n$:

$$s_{n^2-n+2}, \ldots, s_{n^2}, s_1; s_2, \ldots, s_n, s_{n+1}; s_{n+2}, \ldots, s_{2n}, s_{2n+1}$$

Row $2n+1$:

$$s_3, \ldots, s_{n^2+2}; s_{n^2+3}, \ldots, s_{2n^2+2}; s_{2n^2+3}, \ldots, s_{n^2+2}, s_1, s_2$$

and so on.

Replacing the set $S$ by any permutation of its symbols, which we’ll note by $S'$, and applying the same procedure as above, we will obtain a new solution.

The classical Sudoku is obtained for $n = 3$.

Below is an example of this group-permutation algorithm for the classical case:

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For a $4^2 \times 4^2$ square we use the following 16 symbols:


and use the same group-permutation algorithm to solve this Sudoku.

From one solution to the generalized Sudoku we can get more solutions by simply doing permutations of columns or/and of rows of the first solution.
### References:


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