Uniform Redistribution Rule

Let’s consider a finite and discrete frame of discernment \( \Theta \), its hyper-power set \( G^\Theta \) (i.e. Dedekind’s lattice) and two quantitative basic belief assignments \( m_1(.) \) and \( m_2(.) \) defined on \( G^\Theta \) expressed by two independent sources of evidence.

The Uniform Redistribution Rule (URR) consists in redistributing the total conflicting mass \( k_{12} \) to all focal elements of \( G^\Theta \) generated by the consensus operator. This way of redistributing mass is very simple and URR is different from Dempster’s rule of combination [3], because Dempster’s rule redistributes the total conflict proportionally with respect to the masses resulted from the conjunctive rule of non-empty sets. PCR5 and PCR4 [5] do proportional redistributions of partial conflicting masses to the sets involved in the conflict. Here it is the URR formula for two sources: \( \forall A \neq \emptyset \), one has

\[
m_{12URR}(A) = m_{12}(A) + \frac{1}{n_{12}} \sum_{X_1,X_2 \in G^\Theta : X_1 \cap X_2 = \emptyset} m_1(X_1)m_2(X_2)
\]  

(1)
where $m_{12}(A)$ is the result of the conjunctive rule applied to belief assignments $m_1(.)$ and $m_2(.)$, and $n_{12} = Card\{Z \in G^\Theta, m_1(Z) \neq 0 \text{ or } m_2(Z) \neq 0\}$.

For $s \geq 2$ sources to combine: $\forall A \neq \emptyset$, one has

$$m_{12...sURR}(A) = m_{12...s}(A) + \frac{1}{n_{12...s}} \sum_{X_1 \cap X_2 \cap \ldots \cap X_s = \emptyset} \prod_{i=1}^{s} m_i(X_i)$$

(2)

where $m_{12...s}(A)$ is the result of the conjunctive rule applied to $m_i(.)$, for all $i \in \{1, 2, \ldots, s\}$ and

$$n_{12...s} = Card\{Z \in G^\Theta, m_1(Z) \neq 0 \text{ or } m_2(Z) \neq 0 \text{ or } \ldots \text{ or } m_s(Z) \neq 0\}$$

As alternative, we can also consider the cardinal of the ensemble of sets whose masses resulted from the conjunctive rule are non-null, i.e. the cardinality of the core of conjunctive consensus:

$$n_{12...s}^c = Card\{Z \in G^\Theta, m_{12...s}(Z) \neq 0\}$$

We denote this modified version of URR as MURR in the sequel.

3 Example for URR and MURR

Example for URR: Let’s consider $\Theta = \{A, B, C\}$ with the DSm hybrid model as shown on the Figure 1. In

![Figure 1: Hybrid model for $\Theta = \{A, B, C\}$](image)

this hybrid model $C \cap (A \cup B) = \emptyset$ (therefore $A \cap C = \emptyset$ and $B \cap C = \emptyset$). We consider also the following two belief assignments

$$m_1(A) = 0.4 \quad m_1(B) = 0.2 \quad m_1(A \cup B) = 0.4$$

$$m_2(A) = 0.2 \quad m_2(C) = 0.3 \quad m_2(A \cup B) = 0.5$$

then the conjunctive operator provides for this DSm hybrid model a consensus on $A$, $B$, $C$, $A \cup B$, and $A \cap B$ with supporting masses

$$m_{12}(A) = 0.36 \quad m_{12}(B) = 0.10 \quad m_{12}(A \cup B) = 0.20 \quad m_{12}(A \cap B) = 0.04$$

and partial conflicts between two sources on $A \cap C$, $B \cap C$ and $C \cap (A \cup B)$ with

$$m_{12}(A \cap C) = 0.12 \quad m_{12}(B \cap C) = 0.06 \quad m_{12}(C \cap (A \cup B)) = 0.12$$

Then with URR, the total conflicting mass

$$m_{12}(A \cap C) + m_{12}(B \cap C) + m_{12}(C \cap (A \cup B)) = 0.12 + 0.06 + 0.12 = 0.30$$

is uniformly (i.e. equally) redistributed to $A$, $B$, $C$ and $A \cup B$ because the sources support only these propositions. That is $n_{12} = 4$ and thus $0.30/n_{12} = 0.075$ is added to $m_{12}(A)$, $m_{12}(B)$, $m_{12}(C)$ and $m_{12}(A \cup B)$ with URR. One finally gets:

$$m_{12URR}(A) = m_{12}(A) + \frac{0.30}{n_{12}} = 0.36 + 0.075 = 0.435$$

$$m_{12URR}(B) = m_{12}(B) + \frac{0.30}{n_{12}} = 0.10 + 0.075 = 0.175$$

$$m_{12URR}(C) = m_{12}(C) + \frac{0.30}{n_{12}} = 0.00 + 0.075 = 0.075$$
m_{12U}RR(A ∪ B) = m_{12}(A ∪ B) + \frac{0.30}{m_{12}} = 0.20 + 0.075 = 0.275
while the others remain the same. That is m_{12U}RR(A ∩ B) = 0.4. Of course, one has also
m_{12U}RR(A ∩ C) = m_{12U}RR(B ∩ C) = m_{12U}RR(C ∩ (A ∪ B)) = 0

Example for MURR: Let’s consider the same frame, same model and same bba as in previous example. In this case the total conflicting mass 0.30 is uniformly redistributed to the sets A, B, A ∪ B, and A ∩ B only, i.e. to the sets whose masses, after applying the conjunctive rule to the given sources, are non-zero. Thus m_{12} = 4, and 0.30/4 = 0.075. Hence:

\[ m_{12U}RR(A) = 0.36 + 0.075 = 0.435 \]
\[ m_{12U}RR(B) = 0.10 + 0.075 = 0.175 \]
\[ m_{12U}RR(A ∪ B) = 0.20 + 0.075 = 0.275 \]
\[ m_{12U}RR(A ∩ B) = 0.04 + 0.075 = 0.115 \]

4 Partially Uniform Redistribution Rule

It is also possible to do a uniformly partial redistribution, i.e. to uniformly redistribute the conflicting mass only to the sets involved in the conflict. For example, if m_{12}(A ∩ B) = 0.08 and A ∩ B = 0, then 0.08 is equally redistributed to A and B only, supposing A and B are both non-empty, so 0.04 assigned to A and 0.04 to B.

∀A ≠ ∅, one has the Partially Uniform Redistribution Rule (PURR) for two sources

\[ m_{12PU}RR(A) = m_{12}(A) + \frac{1}{2} \sum_{X_1,X_2 ∈ G^o \atop X_1 ∩ X_2 = ∅} m_1(X_1)m_2(X_2) \]

where \( m_{12}(A) \) is the result of the conjunctive rule applied to belief assignments \( m_1(.) \) and \( m_2(.) \).

For \( s \geq 2 \) sources to combine: ∀A ≠ ∅, one has

\[ m_{12...sPU}RR(A) = m_{12...s}(A) + \frac{1}{s} \sum \text{Card}_A(\{X_1,...,X_s\}) \prod_{i=1}^{s} m_i(X_i) \]

where \( \text{Card}_A(\{X_1,...,X_s\}) \) is the number of A’s occurring in \( \{X_1,X_2,...,X_s\} \).

If A = ∅, \( m_{12PU}RR(A) = 0 \) and \( m_{12...sPU}RR(A) = 0 \).

5 Example for PURR

Let’s take back the example of section 3. Based on PURR, \( m_{12}(A ∩ C) = 0.12 \) is redistributed as follows: 0.06 to A and 0.06 to C; \( m_{12}(B ∩ C) = 0.06 \) is redistributed as follows: 0.03 to B and 0.03 to C; and \( m_{12}(C ∩ (A ∪ B)) = 0.12 \) is redistributed in this way: 0.06 to C and 0.06 to A ∪ B. Therefore we finally get

\[ m_{12PU}RR(A) = m_{12}(A) + \frac{0.12}{2} = 0.36 + 0.06 = 0.42 \]
\[ m_{12PU}RR(B) = m_{12}(B) + \frac{0.06}{2} = 0.10 + 0.03 = 0.13 \]
\[ m_{12PU}RR(C) = m_{12}(C) + \frac{0.12}{2} + \frac{0.06}{2} + \frac{0.12}{2} = 0.15 \]
\[ m_{12PU}RR(A ∪ B) = m_{12}(A ∪ B) + \frac{0.12}{2} = 0.20 + 0.06 = 0.26 \]
while the others remain the same. That is \( m_{12PU}RR(A ∩ B) = 0.04 \). Of course, one has also

\[ m_{12PU}RR(A ∩ C) = m_{12PU}RR(B ∩ C) = m_{12PU}RR(C ∩ (A ∪ B)) = 0 \]
6 Neutrality of vacuous belief assignment

Both URR (with MURR included) and PURR are commutative and quasi-associative, and they verify the neutrality of Vacuous Belief Assignment (VBA): since any bba $m_1(.)$ combined with the VBA defined on any frame $\Theta = \{\theta_1, \ldots, \theta_n\}$ by $m_{VBA}(\theta_1 \cup \ldots \cup \theta_n) = 1$, using the conjunctive rule, gives $m_1(\cdot)$, so no conflicting mass is needed to transfer.

7 Conclusion

Two new simple rules of combination have been presented in the framework of DSmT which have a lower complexity than PCR5. These rules are very easy to implement but from a theoretical point of view remain less precise in their transfer of conflicting beliefs since they do not take into account the proportional redistribution with respect to the mass of each set involved in the conflict. So we cannot reasonably expect that URR or PURR outperforms PCR5 but they may hopefully appear as good enough in some specific fusion problems when the level of total conflict is not important. PURR does a more refined redistribution that URR and MURR but it requires a little more calculation.

References


