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Lagrange Multipliers and Neutrosophic Nonlinear Programming Problems Constrained by Equality Constraints

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Abstract: Operations research science is defined as the science that is concerned with applying scientific methods to complex problems in managing and directing large systems of people, including resources and tools in various fields, private and governmental work, peace and war, politics, administration, economics, planning and implementation in various domains. It uses scientific methods that take the language of mathematics as a basis for it and uses computer, without which it would not have been possible to achieve numerical solutions to the raised problems, those that need correct solutions, when the solutions abound and the options are multiple, so we need a decision based on correct scientific foundations and takes into account all the circumstances and changes that you can encounter the decision-maker during the course of work, and nothing is left to chance or luck, but rather everything that enters into the account and plays its role in decision-making, and we get that when we use the concepts of neutrosophic science to reformulate what the science of operations research presented in terms of methods and methods to solve many practical problems, so we will present in this research a study aimed at shedding light on the most important methods used to solve nonlinear models, which is the Lagrangian multiplier method for nonlinear models constrained by equality and then reformulated using the concepts of neutrosophic science.

Keywords: Operations Research; Nonlinear Programming; Lagrange Multiplier; Neutrosophic Science; Neutrosophic Nonlinear Programming; Lagrange Neutrosophic Multiplier.

1. Introduction

Science is the basis for managing life affairs and human activities. Operations research science is the applied aspect of mathematics. It is one of the most important modern sciences that are concerned with practical issues and meet the desire and demand of decision-makers to obtain ideal decisions through the methods it presented that are appropriate for all issues, meaning that it depends on the attached data. In every issue, these data are values that have been developed through observation, and their accuracy depends on who collects them. They are also specific for a certain period that may not be appropriate for subsequent circumstances. Therefore, they are inaccurate data that are not fully confirmed. Before the advent of neutrosophic science, we used to rely on this data. To study factual issues and accept the results as they are, but after the neutrosophic science that made a great revolution in all fields. Its ability to produce optimal results upon which ideal decisions can be built that take into account all circumstances. Many researchers interested in scientific development tended to reformulate many scientific concepts using this science, so we have neutrosophic numbers - neutrosophic groups - neutrosophic probabilities - neutrosophic statistics - neutrosophic differentiation - neutrosophic integration - neutrosophic linear programming - neutrosophic dynamic programming - neutrosophic simulation, etc. and many more [1-28].

The method of nonlinear programming is one of the most important methods presented by the science of operations research because most of the practical issues devolve into nonlinear models. Therefore, in previous research, we formulated some basic concepts of nonlinear programming using the concepts of neutrosophy [26]. In addition to what we have done earlier, in this research we will paraphrase the Lagrangian multiplication method for nonlinear models constrained by equal constraints using the concepts of neutrosophic science, by taking the data of the issue under study, neutrosophic numbers It has the following standard form N = a+bI where real or composite coefficients, represents a and baspecified part fbI The indefinite part of the numberNindeterminacy and could be $[\lambda_1 , \lambda_2]$or$\{\lambda_1 , \lambda_1\}$ or something else is any set close to the true value a.

2. Discussion

We know that if the nonlinear programming issue consists of only a target function and the target function is convex or (concave), then there is a single optimal solution at a point where all the derivatives are non-existent. The only optimal, but in most realistic issues the goal is to find the maximum or minimum value of a target function that is subject to several restrictions. To calculate one of the variables in terms of the rest of the variables, then replace it with the objective function statement, thus obtaining a new objective function and a new unrestricted example problem. We will use this technique as mentioned in the references [29,30].

To convert nonlinear restricted neutrosophic problems into nonlinear non-restricted neutrosophic problems, and for that, the following information taken from the references must be recalled [27, 28]:

$$f_N'(x) = \lim_{h\to 0} \frac{[\inf f(x+h) - inf f(x), sup f(x+h) - sup f(x)]}{[inf H, sup H]}$$

This definition is a generalization of the traditional derivative definition.

Text of the constrained neutrosophic nonlinear programming problem:

Based on what is stated in the reference [29], we offer the following:

If it is required to find the (maximum or minimum), value of a function Neutrosophic is continuous, derivable $\boldsymbol{y_0 = f_N(x_1, x_2, ---, x_n)}$ and subject to the neutrosophic constraint $\boldsymbol{g_N(x_1, x_2, ---, x_n) = \alpha}$ Also this function is continuous and derivable.

1. We choose the variable $\boldsymbol{x_n}$ in the constraint and express it with the remaining $\boldsymbol{n-1}$ variables as follows:

$$\boldsymbol{x_n = H_N(x_1, x_2, ---, x_{n-1})}$$

Then we substitute the target function we get

$$\boldsymbol{y_0 = \overline{f_N}\big(x_1, x_2, ---, x_{n-1}, H_N(x_1, x_2, ---, x_{n-1})\big)}$$

Thus, the problem turned into an unrestricted one. Traditional methods can be used to obtain a maximum or minimum limit because the necessary and sufficient condition for the boundary points is that the first derivatives do not exist:

$$\frac{\partial y_0}{\partial \boldsymbol{x_j}} = 0 \quad ; j = 1,2, ---(n-1)$$

Using the chain rule, we get the following:

$$\frac{\partial y_0}{\partial \boldsymbol{x_j}} = \frac{\partial \overline{\boldsymbol{f_N}}}{\partial \boldsymbol{x_j}} + \frac{\partial \overline{\boldsymbol{f_N}}}{\partial \boldsymbol{x_n}} \cdot \frac{\partial \boldsymbol{H_N}}{\partial \boldsymbol{x_j}} \quad ; j = 1,2,---(n-1)$$

Since the $\boldsymbol{g_N}(\boldsymbol{x_1}, \boldsymbol{x_2}, ---, \boldsymbol{x_n}) = \boldsymbol{\alpha}$
We get

$$\frac{\partial \boldsymbol{g_N}}{\partial \boldsymbol{x_j}} + \frac{\partial \boldsymbol{g_N}}{\partial \boldsymbol{x_n}} \cdot \frac{\partial \boldsymbol{H_N}}{\partial \boldsymbol{x_j}} = 0 \quad ; j = 1,2,---(n-1)$$

$$\Rightarrow \frac{\partial \boldsymbol{H_N}}{\partial \boldsymbol{x_j}} = -\frac{\partial \boldsymbol{g_N}}{\partial \boldsymbol{x_j}} \Big/ \frac{\partial \boldsymbol{g_N}}{\partial \boldsymbol{x_n}} \quad ; j = 1,2,---(n-1)$$

On condition: $\frac{\partial g_N}{\partial x_n} \neq 0$
Then we find

$$\frac{\partial y_0}{\partial \boldsymbol{x_j}} = \frac{\partial \overline{\boldsymbol{f_N}}}{\partial \boldsymbol{x_j}} - \left[\frac{\partial \overline{\boldsymbol{f_N}}}{\partial \boldsymbol{x_n}} \cdot \frac{\partial \boldsymbol{g_N}}{\partial \boldsymbol{x_j}} \Big/ \frac{\partial \boldsymbol{g_N}}{\partial \boldsymbol{x_n}} \right] = 0 \quad ; j = 1,2,---(n-1)$$

If the resulting solution vector is the vector that achieves a (maximum or minimum) value, then$(x_{N1}^*, x_{N2}^*, ---, x_{Nn}^*)$, They are the values that make the function maximum or minimum.
We symbolize λ_N the following:

$$\lambda_N = \frac{\partial \boldsymbol{f_N}}{\partial \boldsymbol{x_n}} \Big/ \frac{\partial \boldsymbol{g_N}}{\partial \boldsymbol{x_n}}$$

So it is:

$$\frac{\partial \overline{\boldsymbol{f_N}}}{\partial \boldsymbol{x_j}} - \lambda_N \frac{\partial \boldsymbol{g_N}}{\partial \boldsymbol{x_j}} = 0 \quad ; j = 1,2,--,n$$

The supporting condition is:

$$\boldsymbol{g_N}(\boldsymbol{x_1}, \boldsymbol{x_2}, ---, \boldsymbol{x_n}) = \boldsymbol{\alpha}$$

And so we have obtained $n+1$ Equation and $n+1$ unknown, these conditions are necessary for an optimal solution, provided that all derivatives do not exist $\frac{\partial g_N}{\partial x_j} \neq 0$

when$(x_{N1}^*, x_{N2}^*, ---, x_{Nn}^*)$
From the above, we can write the following:

$$\boldsymbol{y_0} = \overline{\boldsymbol{f_N}}(\boldsymbol{x_1}, \boldsymbol{x_2}, ---, \boldsymbol{x_n}) - \lambda_N [\boldsymbol{g_N}(\boldsymbol{x_1}, \boldsymbol{x_2}, ---, \boldsymbol{x_n}) - \boldsymbol{\alpha}]$$

Then, we calculate the derivatives:

$$\frac{\partial y_0}{\partial \boldsymbol{x_j}} = \frac{\partial \boldsymbol{f_N}}{\partial \boldsymbol{x_j}} - \lambda_N \frac{\partial \boldsymbol{g_N}}{\partial \boldsymbol{x_j}} = 0$$

$$\frac{\partial y_0}{\partial \lambda} = -[\boldsymbol{g_N}(\boldsymbol{x_1}, \boldsymbol{x_2}, ---, \boldsymbol{x_n}) - \boldsymbol{\alpha}]$$

2.1 Example

This example is shown in reference [29] (values are dependent and constraints are classic values)

2.1.1. Find the minimum value of the function

$$f(x) = 3x_1^2 + x_2^2 + 2x_1x_2 + 6x_1 + 2x_2$$

Subject to constraint

$$2x_1 - x_2 = 4$$

The solution

We form the Lagrangian function:

$$L(x,\lambda) = 3x_1^2 + x_2^2 + 2x_1x_2 + 6x_1 + 2x_2 - \lambda(2x_1 - x_2 - 4)$$

The following optimal solution was obtained:

$$x_1^* = \frac{7}{11} \quad , x_2^* = \frac{-30}{11} \quad , \quad \lambda^* = \frac{24}{11}$$

And by testing the function using the Hessian matrix, it was found that this function is a convex function and the constraint is convex

Then the radius of the solution that we obtained is a minimum limit and the value of the function is:

$$f_N^*(x_1^*, x_2^*) = 3.55$$

As we mentioned earlier, if some of the values in the target function or constraints are undefined, ambiguous, or uncertain values, then the issue becomes a neutrosophic issue clearly)

In the previous example, we will take the coefficients x_1^2 uncertain (All values can be undefined; we will suffice with one value to convey the idea of using neutrosophic numbers clearly

2.1.2 Find the minimum value of the neutrosophic function.

$$f(x) = \{2,3,4\}x_1^2 + x_2^2 + 2x_1x_2 + 6x_1 + 2x_2$$

Subject to constraint

$$2x_1 - x_2 = 4$$

The solution

We form the Lagrangian function:

$$L(x,\lambda) = \{2,3,4\}x_1^2 + x_2^2 + 2x_1x_2 + 6x_1 + 2x_2 - \lambda(2x_1 - x_2 - 4)$$

$$\frac{\partial L}{\partial x_1} = \{4,6,8\}x_1 + 2x_2 + 6 - 2\lambda = 0$$

$$\frac{\partial L}{\partial x_2} = 2x_1 + 2x_2 + 2 + \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = 2x_1 - x_2 + 4 = 0$$

We solve the following set of equations:

$$\{4,6,8\}x_1 + 2x_2 + 6 - 2\lambda = 0$$

$$2x_1 + 2x_2 + 2 + \lambda = 0$$

$$2x_1 - x_2 + 4 = 0$$

We get

We obtained the system of three equations with three unknowns. By solving this system, we get:

$$x_1^* = \left\{\frac{7}{10}, \frac{7}{11}, \frac{7}{12}\right\} \quad , x_2^* = \left\{\frac{-13}{5}, \frac{-30}{11}, \frac{-17}{6}\right\} \quad , \quad \lambda^* = \left\{\frac{9}{5}, \frac{24}{11}, \frac{15}{6}\right\}$$

They are neutrosophic values, and the value of the target function is as follows:

$$f_N^*(x_1^*, x_2^*) = \{3.1, 3.55, 3.24\}$$

That is, we got the optimal solution vector, which is a neutrosophic value. An extreme of a subordinate, it can be a minor end or a major end. To determine its type we will use the test given in reference [26].

1. The constraint is a linear function, as it is convex and concave at the same time.
2. To specify the type of follower $f(x)$(convex or concave) we resort to the Hessian matrix for this function:

$$H_N(x) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$$

$$H_N(x) = \begin{bmatrix} \{4, 6, 8\} & 2 \\ 2 & 2 \end{bmatrix}$$

The matrix is symmetric and the main diagonal elements are also positive.
Major basic minor determinants are positive because:

$$|\{4, 6, 8\}| > 0$$

$$\begin{vmatrix} \{4, 6, 8\} & 2 \\ 2 & 2 \end{vmatrix} = \{8, 12, 16\} - 4 = \{4, 8, 12\} > 0$$

Hence the Hessian matrix of the function$f_N(x)$Positive knowledge, that is, the function is convex. From the above, the Vector of the solution that we obtained is a minor limit of the value neutrosophic to follow him:

$$f_N^*(x_1^*, x_2^*) = \{3.1, 3.55, 3.24\}$$

3. Conclusions

In the previous study, we presented an important technique for solving neutrosophic nonlinear models constrained by equal constraints. Through the study, we found that there is a difference in the values of the optimal solution when using the technique and the data are traditional values and using them and the data are neutrosophic values, since the goal of solving examples problems is to find the maximum value that expresses profit or profitability and the smallest value that expresses the amount of loss or cost for a follower of a goal within certain constraints, and since mathematical models are built using data collected on the case under study, and these data are values that represent the current reality of the work environment, and any change in the surrounding conditions leads to a change in the results of the solution, which may cause unexpected losses whose nature is determined by the type of issue under study, it may be human, material, or otherwise, so we focus on the need to use neutrosophic values when collecting data for any realistic issue, values that take into account the worst conditions to the best, as we focus on necessity reformulation of many other techniques for solving nonlinear models using neutrosophic concepts such as projection gradients and vibration–Newton's method–Fibonacci Search.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

References

1. Maissam jdid, Rafif Alhabib, and AA Salama ,The static model of inventory management without a deficit with Neutrosophic logic, International Journal of Neutrosophic Science, Vol. 16, 2021.
2. Maissam Jdid, AA Salama, Huda E Khalid ,Neutrosophic Handling of the Simplex Direct Algorithm to Define the Optimal Solution in Linear Programming, International Journal of Neutrosophic Science, Vol.18,No. 1, 2022
3. Maissam Ahmed Jdid _A. A. Salama _Rafif Alhabib _Huda E. Khalid_Fatima Suliman ,Neutrosophic Treatment of the Static Model of Inventory Management with Deficit, International Journal of Neutrosophic Science, Vol.18,No. 1, 2022
4. Maissam Jdid, Huda E Khalid, Mysterious Neutrosophic Linear Models, International Journal of Neutrosophic Science, Vol.18,No. 2, 2022
5. Maissam Jdid, Rafif Alhabib, Neutrosophical dynamic programming, International Journal of Neutrosophic Science, Vol.18, No.3, 2022
6. Maissam Jdid, Basel Shahin, Fatima Suliman, Important Neutrosophic Rules for Decision – Making in Case of Uncertain Data, International Journal of Neutrosophic Science, Vol.18, No.3, 2022
7. Maissam jdid, Rafif Alhabib, and AA Salama, Fundamentals of Neutrosophical Simulation for Generating Random Numbers Associated with Uniform Probability Distribution, Neutrosophic sets and Systems , Vol 49,2022
8. Maissam Jdid , Rafif Alhabib , Ossama Bahbouh , AA Salama , Huda E. Khalid ,The Neutrosophic Treatment for Multiple Storage Problem of Finite Materials and Volumes, International Journal of Neutrosophic Science (IJNS) Vol. 18, no. 1, 2022
9. Maissam Jdid, Rafif Alhabib, Huda E. Khalid, AA Salama The Neutrosophic Treatment of the Static Model for the Inventory Management with Safety Reserve ,International Journal of Neutrosophic Science (IJNS) Vol. 18, no. 2, 2022
10. Maissam Jdid, Huda E Khalid, An Investigation in the Initial Solution for Neutrosophic Transportation Problems (NTP), Neutrosophic sets and Systems NSS , Vol.50,2022
11. Maissam Jdid, Huda E. Khalid, Neutrosophic Mathematical formulas of Transportation Problems, Neutrosophic sets and Systems, NSS, Vol.51,2022
12. Maissam jdid- Fatima AlSuliman, A study in Integral of sine and Cosine Functions, Journal of Mathematical Structures and Applications, (GJMSA), Vol.2, No. 1, 2022
13. Maissam jdid ,Important Neutrosophic Economic Indicators of the Static Model of Inventory Management without Deficit, Journal of Neutrosophic and Fuzzy Systems (JNFS), Vo.5, No.1, 2023
14. Maissam jdid- Hla Hasan, The state of Risk and Optimum Decision According to Neutrosophic Rules, International Journal of Neutrosophic Science (IJNS), Vol. 20, No. 1, 2023

15. Maissam Jdid, Rafif Alhabib, AA Salam The Basics of Neutrosophic Simulation for Converting Random Numbers Associated with a Uniform Probability Distribution into Random Variables Follow an Exponential Distribution ,Journal Neutrosophic Sets and Systems ,NSS, Vol.53,2023
16. Mohammed Alshikho, Maissam Jdid, Said Broumi, Artificial Intelligence and Neutrosophic Machine learning in the Diagnosis and Detection of COVID 19, Journal Prospects for Applied Mathematics and Data Analysis, Vol 01, No, 02 USA, 2023
17. Mohammed Alshikho, Maissam Jdid, Said Broumi, A Study of a Support Vector Machine Algorithm with an Orthogonal Legendre Kernel According to Neutrosophic logic and Inverse Lagrangian Interpolation, Journal of Neutrosophic and Fuzzy Systems (JNFS), Vo.5, No.1, 2023
18. Maissam Jdid, Khalifa Alshaqsi, Optimal Value of the Service Rate in the Unlimited Model , Journal of Neutrosophic and Fuzzy Systems (JNFS), Vo.6, No.1, 2023 $\mathbf{M}\backslash \boldsymbol{M}\backslash \mathbf{1}$
19. Maissam Jdid, AA Salam, Using the Inverse Transformation Method to Generate Random Variables that follow the Neutrosophic Uniform Probability Distribution. Journal of Neutrosophic and Fuzzy Systems (JNFS), Vo.6, No.2, 2023
20. Florentin Smarandache, Maissam Jdid, On Overview of Neutrosophic and Plithogenic Theories and Applications, Applied Mathematics and Data Analysis, Vo.2, No.1, 2023
21. Maissam Jdid, Neutrosophic Mathematical Model of Product Mixture Problem Using Binary Integer Mutant, Journal of Neutrosophic and Fuzzy Systems(JNFS), Vo.6, No.2, 2023
22. Maissam Jdid, The Use of Neutrosophic linear Programming Method in the Field of Education, Handbook of Research on the Applications of Neutrosophic Sets Theory and Their Extensions in Education, Chapter 15, IGI-Global, 2023
23. Maissam Jdid , Florentin Smarandache , Said Broumi, Inspection Assignment Form for Product Quality Control, Journal of Neutrosophic Systems with Applications, Vol. 1, 2023
24. Maissam Jdid, Said Broumi, Neutrosophic Rejection and Acceptance Method for the Generation of Random Variables, Neutrosophic Sets and Systems, NSS, Vol.56, 2023
25. Maissam Jdid , Florentin Smarandache, The Use of Neutrosophic Methods of Operation Research in the Management of Corporate Work, , Journal of Neutrosophic Systems with Applications, Vol. 3, 2023
26. Maissam Jdid, Neutrosophic Nonlinear Models, Journal Prospects for Applied Mathematics and Data Analysis, Vo.2, No.1, 2023
27. Florentin Smarandache, Introduction to Neutrosophic Measure, Neutrosophic Integral, and Neutrosophic Probability, SciTech-Education Publisher, Craiova - Columbus, 2013.
28. Neutrosophy. / Neutrosophic Probability, Set, and Logic, by F. Smarandache, Amer. Res. Press, Rehoboth, USA, 105 p., 1998;
29. Bakaja Ji .S , Mualla, W... and others - Operations Research Book, translated into Arabic - The Arab Center for Arabization, Translation, Authoring and Publishing -Damascus -1998. (Arabic version).
30. Maissam Jdid, Discrete optimization control problem, https://www.elibrary.ru/item.asp?id=19181958,2002.

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