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Research Paper:

Novel Concepts on Domination in Neutrosophic Incidence Graphs with Some Applications

Siti Nurul Fitriah Mohamad[*,**], **Roslan Hasni**[**,†], **and Florentin Smarandache**[***]

[*]College of Computing, Informatics and Mathematics, Universiti Teknologi MARA
Machang, Kelantan 18500, Malaysia
E-mail: fitriah@uitm.edu.my
[**]Special Interest Group on Modeling and Data Analytics,
Faculty of Ocean Engineering Technology and Informatics, Universiti Malaysia Terengganu
Kuala Nerus, Terengganu 21030, Malaysia
E-mail: hroslan@umt.edu.my
[***]Mathematics & Science Department, The University of New Mexico
705 Gurley Avenue, Gallup, New Mexico 87301, USA
E-mail: fsmarandache@gmail.com
[†]Corresponding author
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In graph theory, the concept of domination is essential in a variety of domains. It has broad applications in diverse fields such as coding theory, computer network models, and school bus routing and facility location problems. If a fuzzy graph fails to obtain acceptable results, neutrosophic sets and neutrosophic graphs can be used to model uncertainty correlated with indeterminate and inconsistent information in arbitrary real-world scenario. In this study, we consider the concept of domination as it relates to single-valued neutrosophic incidence graphs (SVNIGs). Given the importance of domination and its utilization in numerous fields, we propose the application of dominating sets in SVNIG with valid edges. We present some relevant definitions such as those of valid edges, cardinality, and isolated vertices in SVNIG along with some examples. Furthermore, we also show a few significant sets connected to the dominating set in an SVNIG such as independent and irredundant sets. We also investigate a relationship between the concepts of dominating sets and domination numbers as well as irredundant and independence sets in SVNIGs. Finally, a real-life deployment of domination in SVNIGs is investigated in relation to COVID-19 vaccination locations as a practical application.

Keywords: dominating set, domination number, neutrosophic graph, single-valued neutrosophic incidence graph

1. Introduction

In recent decades, graph theory has been increasingly considered important as a branch of applied mathematics, generally referred to as combinatorics. In fields as diverse as topology, number theory, algebra, optimization, and geometry, graphs are commonly utilized as a tool for addressing combinatorial issues, and their importance in computer science is well known. Weighted graphs represent the strength of links between the vertices, and can be utilized to define any model that consists of points and lines. The concept of domination was first established for the game of chess in the 1850s. Several chess players in Europe debated the matter of deciding the least number of queens that could be substituted on a chessboard to ensure that all of the squares were occupied by a queen. As a solution to this problem, Ore [1] and Berge [2] first proposed the concept of domination in graphs in 1962.

In graph theory, domination may be utilized to model a wide variety of systems, such as in abstract formalizations like the facility location problem, social network analysis, and problems in matching and coding theory, as well as the operation of real technologies such as communication networks, security systems, clutters, and block cutters. For example, it aims to address problems about facility location problems in which the number of facilities such as police stations, fire stations, hospitals, and supermarkets, and to shorten the routes that travelers must take to reach the nearest facility. In abstract terms, if the maximum distance to a facility is defined and efforts to lower the number of facilities required to accommodate everyone are made, similar situations may be expected to develop. Furthermore, the concept of domination has been considered in applications such as in monitoring communication or electrical networks, finding sets of representatives, and surveying land.

If vertices are considered to symbolize different cities, edges can indicate roads that link them. The result of this arrangement is a fuzzy graph (FG) that depicts the volume of traffic from one city to another. Here, the city with the most residents will have the most entrance ramps. If A and B are two cities and AB denotes the road that connects them, we can consider that $(\mathrm{A}, \mathrm{AB})$ can display a ramp system from road AB to city A. Both A and B exhibit a one-to-one influence on AB in unweighted graphs. The impact of A on AB represented by (A,AB) on a directed graph is 1, but (B,AB) is 0. Dinesh [3] presented fuzzy incidence graphs (FIGs) as an expansion of this concept. An FIG depicts the extent of the connections between vertices in a set and the effect of a vertex on a relation pair.

In 1965, the concept of fuzzy sets (FSs) was proposed by Zadeh [4], and subsequently generalized to model fuzziness and uncertainty. Since then, FS theory has been widely explored in a variety of fields. For example, Rosenfeld [5] established FG theory and also presented FGs and many important notions such as fuzzy trees, fuzzy cycles, fuzzy paths, etc., based on Zadeh's notions on the FS. In addition, Kaufmann [6] presented FGs based on Zadeh's fuzzy relation. However, because the features of graph issues are frequently indeterminate and inconsistent, it is considered preferable to handle them utilizing the approach of neutrosophic logic, which provides the system greater precision in comparison to the classic set and FS.

To cope with the difficulty, Smarandache [7] proposed the idea of neutrosophic set (NS) theory as an extension of the classic FSs, as well as that of intuitionistic FSs (IFSs). In NSs, falsity membership functions (F), indeterminacy membership functions (I), and truth membership functions (T) comprise a membership value. Here, each membership value is a non-standard or real-standard subset of the non-standard unit interval $]0^-,1^+[$, and their sum is not restricted. To model real-world issues more easily by using NS, Wang et al. [8] introduced the idea of single-valued NSs (SVNSs), which have three independent components with values in the standard unit interval $[0,1]$.

Considering the concept of neutrosophic graphs (NGs) is highly effective compared to FGs, it expands the range of applications of graph theory across various fields such as in decision-making problems [9–12]. In real-life settings, NGs, together with the theory of NSs and the concept of domination, serve an important role in implying and dealing with inconsistent, indeterminate, and imprecise information. In recent years, the study of domination in the context of NGs has attracted attention as a topic of active research. Hussain et al. [13] studied domination numbers (DNs) in neutrosophic soft graphs. Moreover, Devi [14] introduced minimal domination via neutrosophic over graphs. Mullai and Broumi [15] introduced dominating energy in NGs, and more recently, Khan et al. [16] utilized strong edges to introduce the notions of paired domination, matching, and covering in a single-valued NG (SVNG). To the best of our knowledge, no prior works have analyzed the concept of domination in the context of neutrosophic incidence graphs. Therefore, motivated by [17, 18], in this study, we consider valid edges to expand the concept of domination to single-valued neutrosophic incidence graphs (SVNIGs). The key contributions of this study are summarized as follows:

Table 1. Basic notations.

Abbreviations	Description
SVNIG	Single-valued neutrosophic incidence graph
SVNIDS	Single-valued neutrosophic incidence dominating set
SVNIDN	Single-valued neutrosophic incidence domination number
SVNIIRS	Single-valued neutrosophic incidence irredundant set
SVNIPN	Single-valued neutrosophic incidence valid private neighborhood
SVNIIDS	Single-valued neutrosophic incidence independent set

Table 2. The list of symbols.

Symbols	Description
$\tilde{G}(V,E,I)$	Incidence graph
$\xi(A,B,C)$	SVNIG
δ_{IVN}	Minimum cardinality single-valued neutrosophic incidence valid neighborhood degrees of $\xi(A,B,C)$
Δ_{IVN}	Maximum cardinality single-valued neutrosophic incidence valid neighborhood degrees of $\xi(A,B,C)$
$\gamma_{IDN}(\xi)$	Minimum cardinality of SVNIDS
$\Gamma_{IDN}(\xi)$	Maximum cardinality of the minimal SVNIDS
$ir_N(\xi)$	Minimum cardinality among all maximal SVNIIRSs
$Ir_N(\xi)$	Maximum cardinality among all maximal SVNIIRSs
$\alpha_{IDS}(\xi)$	Minimum cardinality among all maximal SVNIIDS
$\beta_{IDS}(\xi)$	Maximum cardinality among all maximal SVNIIDS

- An original conceptualization of domination on SVNIG based on valid edges is introduced.
- The descriptions of valid edges, cardinality, and isolated vertices in SVNIGs are explained along with instances.
- We consider a few significant sets connected to the dominating set (DS) in SVNIGs, such as independent and irredundant sets. All basic notation is shown in **Table 1**, and the symbols used are shown in **Table 2**.
- We establish a relationship between the concepts of DS and DN as well as those of irredundant and independent sets in the context of SVNIGs.
- Real-life applications of the concept of domination in SVNIGs are discussed.

The remainder of this study is organized as follows. In this introduction, we discussed the historical background of incidence and domination graphs, as well as the concepts of the FSs and NSs. In Section 2, we briefly review the relevant literature on incidence graphs in fuzzy and neutrosophic environments, and present the concepts of domination in FGs as well as in FIGs and NGs. Section 3 offers a brief introduction to graphs and NSs as used in this work. In Section 4, we formulate the concept of domination in SVNIGs and derive some key aspects. In Section 5, we implement the proposed approach to construct a model of the optimal locations for COVID-19 vaccination administration centers (CVACs). Finally, in Section 6, we summarize our findings and conclude with a discussion of the limitations of this work along with some suggested avenues for further research.

2. Literature Review

Brualdi and Massey [19] first presented the concept of an incidence graph in their work on incidence and incidence chromatic numbers. Such incidence graphs can typically be depicted as a triplet, in which one aspect represents a fine set of vertices, a second denotes a finite set of edges, and a third refers to an incidence function. Dinesh [3] further developed the theory for unordered pairs of vertices that are not incident with end vertices. In contrast, fuzzy incidence depicts the relationships between vertices and offers data on the effect of the vertex on edges' influence. Subsequently, Dinesh [20] expanded on the idea of FIGs and introduced some additional concepts related to this body of knowledge.

Furthermore, Mathew and Mordeson [21] also examined concepts of connectivity in FIGs. In interconnected networks with influenced flows, connectivity is crucial. Hence, investigating the connectivity qualities of graphs is important. The theory of FIGs was then explored by Malik et al. [22] to address human trafficking problems. They argued that the importance of this work is its quantification of a country's vulnerability and of governments' actions in response to human trafficking. Similarly, Mathew et al. [23] investigated certain connectivity and incidence cuts properties in FIGs. In graphs modeling human trafficking networks, incidence is used to model flows. Finally, Mathew and Moderson [24] described fuzzy incidence blocks (FIBs) and discussed their application to the problem of illegal migration. They applied FIBs to prevent a network's susceptible linkages from being ignored, utilizing FIGs as a non-deterministic network model with supporting links. Subsequently, Akram et al. [25] extended FIGs to a neutrosophic environment, and presented the concept of SVNIGs and examined their connectivity. Building on this research, Akram et al. [26] investigated the application of bipolar NSs to incidence graphs and developed a few relevant features. Subsequently, the theory of neutrosophic vague incidence graphs (VIGs) was then developed by Hussain et al. [27], who formulated pair, vertex, and edge connectivity in neutrosophic VIGs. Furthermore, Mohamad et al. [28] applied the concept of innovative interval-valued neutrosophic incidence graphs to the safe root travelling problem.

Ore [1] and Berge [2] began researching DSs in graphs in 1962. Cockayne and Hedetniemi [29] subsequently derived irredundant numbers, independent domination, and the concept of domination. A. Somasundaram and S. Somasundaram [30] were the first to establish the concept of domination in FGs. They utilized effective edges to identify domination and total DNs (TDNs) for various classes in FGs. Subsequently, Somasundaram [31] found domination in the product of two graphs and investigated their domination parameter, as well as many FG operations such as Cartesian product, composition, joining, and union. Additionally, Gani and Ahamed [32] pioneered the concepts of weak and strong domination in FGs and described their various characteristics.

Furthermore, Natarajan and Ayyaswamy [33] developed the idea of strong (weak) domination in FGs, explaining several intriguing findings for this new parameter in FGs. In addition, Vimala and Sathya [34] considered FG point set DNs and utilized the FGs neighborhood degrees to acquire some limitations. Besides that, Manjusha and Sunitha [35–40] have provided some proofs on domination in FGs. Moreover, Ponnappan et al. [41] established the concept of the total edge DN, the edge DN, total edge domination, and edge domination for various types of FGs. In addition, Dharmalingam and Rani [42] suggested the concepts of fuzzy equitable independent sets in equitable FGs, fuzzy equitable independent sets, strong (weak) fuzzy equitable DSs, minimal fuzzy equitable DSs, and fuzzy equitable DSs. They presented several remarkable insights for this new parameter. Accordingly, Dharmalingam and Nithya [43, 44] established the concepts of excellent and very excellent domination in FGs. Here, the notion of a DS was proposed by Bozhenyuk et al. [45] as an invariant of an intuitionistic FG (IFG).

Nazeer et al. [46] utilized a new concept of domination in FIGs developed in their previous work and applied it to the choice of an adequate medical lab among a range of laboratories. Then, Nazeer et al. [47] expand on the idea of domination idea by combining FIGs with strong pairings and applying it to diverse nations' trading systems. Subsequently, Afsharmanesh and Borzooei [17] utilized incidence valid edges to establish DSs in FIGs and examined numerous significant sets pertaining to DS, such as irredundant and independent sets. Rao et al. [18] expanded the concept of domination concept in FIGs to VIGs, utilizing valid edges and applying their approach to the optimal locations of COVID-19 testing facilities in another investigation.

3. Preliminaries

In this section, we recall some fundamental definitions with respect to SVNSs, FIGs, and domination. In this study, maximum and minimum operators are represented by $\vee$ or max, and $\wedge$ or min, respectively.

Definition 1— [48]: An FG $G = (\sigma, \mu)$ is a pair of functions together with an underlying set of vertices V and a set of edges E, where $\sigma: V \to [0,1]$ and $\mu: V\times V \to [0,1]$ such that $\mu(uv) \le \sigma(u)\wedge\sigma(v)$, $\forall u,v\in V$.

Definition 2— [31]: Let $G=(\sigma,\mu)$ denote an FG on V with $u,v\in V$. Note that u dominates v in G if $\mu(u,v)=\sigma(u)\wedge\sigma(v)$. A subset S of V is known as a DS in G provided that for every $u\in V-S$ there exists an $u\in S$ given that u dominates v. Meanwhile, a subset S of V is known as a total DS in G provided that for every $u\in V-S$ there exists an $u\in S$ given that u dominates v.

Definition 3— [31]: The minimum fuzzy cardinality of a DS in $G=(\sigma,\mu)$ is known as the DN of FGs $G=(\sigma,\mu)$, which is expressed by γ_f or $\gamma_f(G)$. Meanwhile, the minimum fuzzy cardinality of a total DS in $G=(\sigma,\mu)$ is known as the TDN of FGs $G=(\sigma,\mu)$, which is expressed by γ_{tf} or $\gamma_{tf}(G)$.

Definition 4— [20]: Let graph $G=(V,E)$, where μ and σ are FSs of E and V, respectively. Let $V\times E$ possess an FS δ. Provided that $\delta(v,e)\le\sigma(v)\wedge\mu(e)$ for every $e\in E$ and $v\in V$, then δ is known as the fuzzy incidence of G, while (σ,μ) is referred to as a fuzzy subgraph of G. Let δ be a fuzzy incidence of G. Then, $G=(\sigma,\mu,\delta)$ is known as an FIG of G.

Definition 5— [8]: Consider $\aleph$ as a universal set. An SVNS A in $\aleph$ is denoted by a falsity-membership function $F_A(x)$, an indeterminacy-membership function $I_A(x)$, and a truth-membership function $T_A(x)$. An SVNS A may be expressed as

$$A=\{x,T_A(x),I_A(x),F_A(x)|x\in\aleph\},$$

where $T_A(x),I_A(x),F_A(x)\in[0,1]$ for each x in $\aleph$. Therefore, the sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$ satisfies the condition $0\le T_A(x)+I_A(x)+F_A(x)\le 3$. Now, for an SVNS A in $\aleph$, the triplets $T_A(x),I_A(x),F_A(x)$ are denoted as a single-valued neutrosophic number, resembling a fundamental element in SVNS.

Definition 6— [49, 50]: An SVNG denotes a pair $G=(V,E)$, where V is an SVNS in $\aleph$ expressed as $V=\{v_1,v_2,\ldots,v_n\}$. This is known as the set of vertices denoting a truth-membership value $T_A: V\to[0,1]$, an indeterminacy-membership value $I_A: V\to[0,1]$, and a falsity-membership value $F_A: V\to[0,1]$ provided that $T_A(v_i)+I_A(v_i)+F_A(v_i)\in[0,3]$, for all $v_i\in V$.

For $E\subseteq V\times V=\{(v_i,v_j)|v_i,v_j\in V,\ i,j=1,2,\ldots,n\}$ is referred to as the set of edges that denotes a truth-membership value $T_B: V\times V\to[0,1]$, an indeterminacy-membership value $I_B: V\times V\to[0,1]$, and a falsity-membership value $F_B: V\times V\to[0,1]$ such that

$$T_B(v_i,v_j)\le T_A(v_i)\wedge T_A(v_j),$$
$$I_B(v_i,v_j)\le I_A(v_i)\wedge I_A(v_j),$$
$$F_B(v_i,v_j)\le F_A(v_i)\vee F_A(v_j).$$

This holds provided that

$$T_B(v_i,v_j)+I_B(v_i,v_j)+F_B(v_i,v_j)\in[0,3],$$

for all $v_i,v_j\in E$.

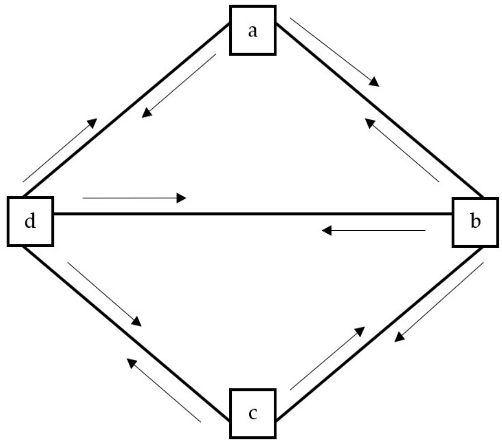

Fig. 1. Incidence graph $\tilde{G}$.

4. Domination in SVNIGs

4.1. Domination in SVNIGs Based on Valid Edges

Definition 7: $\xi=(A,B,C)$ is called an SVNIG of an underlying incidence graph $\tilde{G}=(V,E,I)$ if

$$A=\{\langle T_A(v),I_A(v),F_A(v)\rangle\,|v\in V\},$$
$$B=\{\langle T_B(xy),I_B(xy),F_B(xy)\rangle\,|xy\in E\},$$
$$C=\{\langle T_C(v,xy),I_C(v,xy),F_C(v,xy)\rangle\,|(v,xy)\in I\},$$

such that

$$T_B(xy)\le\min\{T_A(x),T_A(y)\},$$
$$I_B(xy)\le\min\{I_A(x),I_A(y)\},$$
$$F_B(xy)\le\max\{F_A(x),F_A(y)\},$$
$$T_C(v,xy)\le\min\{T_A(v),T_B(xy)\},$$
$$I_C(v,xy)\le\min\{I_A(v),I_B(xy)\},$$
$$F_C(v,xy)\le\max\{F_A(v),F_B(xy)\},\quad\forall v\in V,\ xy\in E,$$

where $T_A,I_A,F_A: V\to[0,1]$ and

$$0\le T_A+I_A+F_A\le 3,$$
$$0\le T_B+I_B+F_B\le 3,$$
$$0\le T_C+I_C+F_C\le 3.$$

Example 1: Let $\tilde{G}=(V,E,I)$ denote an incidence graph as shown in **Fig. 1**, in which $V=\{a,b,c,d\}$, $E=\{ab,bc,bd,cd,ad\}$, and

$$I=\{(a,ab),(b,ab),(b,bc),(c,bc),(b,bd),$$
$$(d,bd),(c,cd),(d,cd),(a,ad),(d,ad)\}.$$

From this, it is clear to indicate that $\xi=(A,B,C)$ is an SVNIG of $\tilde{G}$ as shown in **Fig. 2**, we have

$$A=\{(a,0.1,0.3,0.5),(b,0.2,0.3,0.4),$$
$$(c,0.3,0.4,0.5),(d,0.4,0.6,0.5)\},$$
$$B=\{(ab,0.1,0.3,0.5),(bc,0.2,0.3,0.5),$$
$$(bd,0.2,0.3,0.5),(cd,0.3,0.4,0.5),$$
$$(ad,0.1,0.3,0.5)\},$$
$$C=\{((a,ab),0.1,0.3,0.5),((b,ab),0.1,0.3,0.5),$$
$$((b,bc),0.2,0.3,0.5),((c,bc),0.2,0.3,0.5),$$
$$((b,bd),0.2,0.3,0.5),((d,bd),0.2,0.3,0.5),$$
$$((c,cd),0.3,0.4,0.5),((d,cd),0.3,0.4,0.5),$$
$$((a,ad),0.1,0.3,0.5),((d,ad),0.1,0.3,0.5)\}.$$

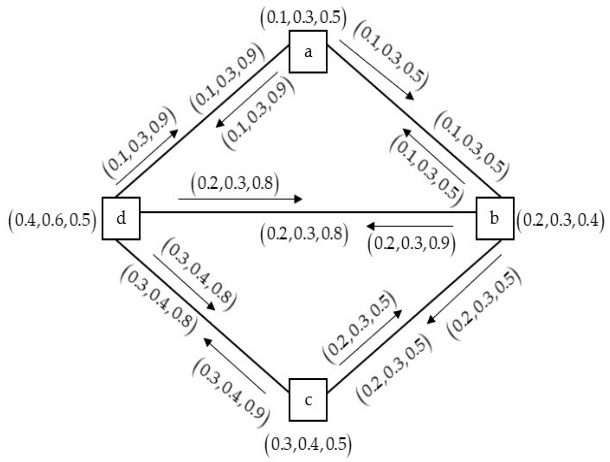

Fig. 2. SVNIG ξ in Examples 1 and 2.

Definition 8: The SVNIG $\xi = (A,B,C)$ support is $\sup(\xi) = (\sup(A), \sup(B), \sup(C))$ such that

$$\sup(A) = \{x | T_A(x) > 0, I_A(x) > 0, F_A(x) > 0\},$$
$$\sup(B) = \{xy | T_B(xy) > 0, I_B(xy) > 0, F_B(xy) > 0\},$$
$$\sup(C) = \{(v,xy) | T_B(v,xy) > 0, I_B(v,xy) > 0, F_B(v,xy) > 0\}.$$

Definition 9: Let $\xi = (A,B,C)$ be denoted as an SVNIG. Thus, $H = (\tilde{A}, \tilde{B}, \tilde{C})$ denotes as an SVNI-subgraph of ξ provided that $\tilde{A} \subseteq A$, $\tilde{B} \subseteq B$, as well as $\tilde{C} \subseteq C$.

Definition 10: Let $\xi = (A,B,C)$ denote an SVNIG. Following from there, a single-valued neutrosophic incidence edge xy of ξ is known as a single-valued neutrosophic incidence valid edge provided that

$$\frac{T_B(xy)}{\min(T_A(x), T_A(y))} \geq 0.5,$$
$$T_C(x,xy) > 0, \quad T_C(y,xy) > 0,$$
$$\frac{I_B(xy)}{\min(I_A(x), I_A(y))} \geq 0.5,$$
$$I_C(x,xy) > 0, \quad I_C(y,xy) > 0,$$
$$\frac{F_B(xy)}{1 + \max(F_A(x), F_A(y))} \leq 0.5,$$
$$F_C(x,xy) > 0, \quad F_C(y,xy) > 0.$$

In another way, it is considered as a single-valued neutrosophic incidence invalid edge.

Definition 11: Consider $\xi = (A,B,C)$ as an SVNIG. Its cardinality is expressed by

$$|\xi| = \left| \sum_{x \in V} \frac{1 + T_A(v) + I_A(v) - F_A(v)}{2} \right| + \left| \sum_{xy \in E} \frac{1 + T_B(xy) + I_B(xy) - F_B(xy)}{2} \right| + \left| \sum_{(v,xy) \in I} \frac{1 + T_C(v,xy) + I_C(v,xy) - F_C(v,xy)}{2} \right|.$$

Also, we have the vertex cardinality $|A|$, edge cardinality $|B|$, and incidence cardinality $|C|$ expressed by

$$|A| = \left| \sum_{x \in V} \frac{1 + T_A(x) + I_A(x) - F_A(x)}{2} \right| = P$$
$$|B| = \left| \sum_{xy \in E} \frac{1 + T_B(xy) + I_B(xy) - F_B(xy)}{2} \right| = Q$$
$$|C| = \left| \sum_{(v,xy) \in I} \frac{1 + T_C(v,xy) + I_C(v,xy) - F_C(v,xy)}{2} \right| = R.$$

Example 2: In **Fig. 2**, it may be clearly observed that ab and bc are single-valued neutrosophic incidence valid edges, from which we obtain

$$\begin{aligned} |A| &= \frac{1+0.1+0.3-0.5}{2} + \frac{1+0.2+0.3-0.4}{2} \\ &\quad + \frac{1+0.3+0.4-0.5}{2} + \frac{1+0.4+0.6-0.5}{2} \\ &= 0.45 + 0.55 + 0.60 + 0.75 \\ &= 2.35, \end{aligned}$$

$$\begin{aligned} |B| &= \frac{1+0.1+0.3-0.5}{2} + \frac{1+0.2+0.3-0.5}{2} \\ &\quad + \frac{1+0.2+0.3-0.8}{2} + \frac{1+0.3+0.4-0.8}{2} \\ &\quad + \frac{1+0.1+0.3-0.9}{2} \\ &= 0.45 + 0.50 + 0.35 + 0.45 + 0.25 \\ &= 2.00, \end{aligned}$$

$$\begin{aligned} |C| &= \frac{1+0.1+0.3-0.5}{2} + \frac{1+0.1+0.3-0.5}{2} \\ &\quad + \frac{1+0.2+0.3-0.5}{2} + \frac{1+0.2+0.3-0.5}{2} \\ &\quad + \frac{1+0.2+0.3-0.8}{2} + \frac{1+0.2+0.3-0.9}{2} \\ &\quad + \frac{1+0.3+0.4-0.8}{2} + \frac{1+0.3+0.4-0.9}{2} \\ &\quad + \frac{1+0.1+0.3-0.9}{2} + \frac{1+0.1+0.3-0.9}{2} \\ &= 0.45 + 0.45 + 0.5 + 0.5 + 0.35 + 0.3 \\ &\quad + 0.45 + 0.4 + 0.25 + 0.25 \\ &= 3.90. \end{aligned}$$

Example 3: Consider an SVNIG $\xi = (A,B,C)$ such that $V = \{e, f, g, h, j\}$, $E = \{ef, fg, gh, hj, ej\}$, and

$$I = \{(e,ef), (f,ef), (f,fg), (g,fg), (g,gh), (h,gh), (h,hj), (j,hj), (e,ej), (j,ej)\},$$

as shown in **Fig. 3**. By routine calculation, the edges ef, fg, and hg are the single-valued neutrosophic incidence valid edges.

Definition 12: The SVNIG $\xi = (A,B,C)$ is considered to be complete provided that

$$\begin{aligned} T_B(xy) &= \min(T_A(x), T_A(y)), \\ I_B(xy) &= \min(I_A(x), I_A(y)), \\ F_B(xy) &= \max(F_A(x), F_A(y)), \\ T_C(v,xy) &= \min(T_A(v), T_B(xy)), \\ I_C(v,xy) &= \min(I_A(v), I_B(xy)), \\ F_C(v,xy) &= \max(F_A(v), F_B(xy)), \\ &\forall v \in V,\ xy \in E. \end{aligned}$$

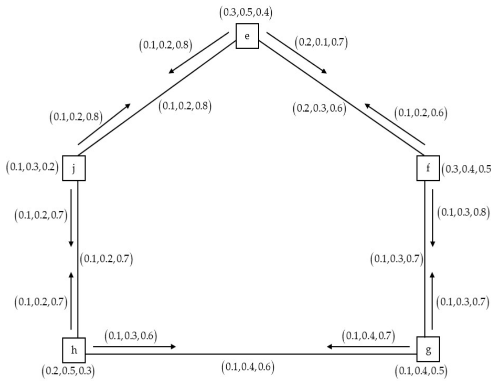

Fig. 3. SVNIG ξ in Example 3.

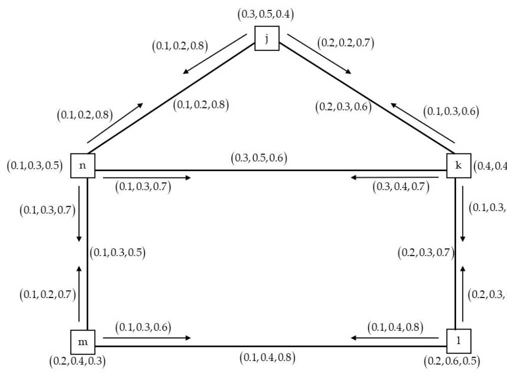

Fig. 4. SVNIG ξ in Examples 4 and 5.

Definition 13: Let $\xi = (A,B,C)$ denote an SVNIG. Thus,

i. a single-valued incidence valid neighborhood of x is expressed by $N_{svniv}(x) = \{y \in V | xy$ is a single-valued neutrosophic incidence valid edge$\}$.

   Note that $N_{svniv}[x] = N_v(x) \cup \{x\}$ denotes a closed single-valued neutrosophic incidence valid neighborhood of x. For non-empty set $D \subseteq V$, we now have

$$N_{svniv}(D) = \{N_{svniv}(x) | x \in D\},$$
$$N_{svniv}[D] = N_{svniv}(D) \cup D.$$

ii. the single-valued neutrosophic incidence valid neighborhood degree of vertex x is expressed by

$$dN_{svniv}(x) = \left(\sum_{y \in N_{svniv}(x)} T_A(y), \sum_{y \in N_{svniv}(x)} I_A(y), \sum_{y \in N_{svniv}(x)} F_A(y) \right).$$

   The maximum and minimum, and cardinality single-valued neutrosophic incidence valid neighborhood degrees of ξ are expressed by Δ_{IVN} and δ_{IVN}, accordingly.

Definition 14: Consider $\xi = (A,B,C)$ to be denoted as an SVNIG. Then, the vertex cardinality of $S \subseteq V$ is expressed as

$$|S| = \left| \sum_{x \in S} \frac{1 + T_A(x) + I_A(x) - F_A(x)}{2} \right|.$$

Example 4: Let SVNIG ξ as shown in **Fig. 4**. Then,

$N_{svniv}(e) = \{f\}$, $N_{svniv}(f) = \{e,g\}$,
$N_{svniv}(g) = \{f,h\}$, $N_{svniv}(h) = \{g\}$,
$N_{svniv}(j) = \emptyset$.

Thus,
$dN_{svniv}(e) = (0.3,0.4,0.5)$, $dN_{svniv}(f) = (0.4,0.9,0.9)$,
$dN_{svniv}(g) = (0.5,0.9,0.8)$, $dN_{svniv}(h) = (0.1,0.4,0.5)$.

Therefore, by routine calculation, it is obvious that $\delta_{IVN} = (0.1,0.4,0.5)$ and $\Delta_{IVN} = (0.5,0.9,0.8)$.

Definition 15: Let $\xi = (A,B,C)$ denote an SVNIG of $\tilde{G}$. Then,

1. $x \in V$ incidentally dominates $y \in V$ in ξ, provided that $y \in N_{svniv}[x]$.

2. a non-empty set of vertices $D \subseteq V$ denotes a single-valued neutrosophic incidence DS (SVNIDS) in an SVNIG ξ, provided that there exists $y \in D$ whenever x incidentally dominates y. In other words, $V = N_{svniv}[D]$, $\forall x \in V - D$.

3. The minimum cardinality of SVNIDS in an SVNIG of $\tilde{G}$ is expressed as a single-valued neutrosophic incidence DN (SVNIDN) of ξ, which is expressed by $\gamma_{IDN}(\xi)$. Clearly, $\gamma_{IDN}(\xi) \leq P$.

4. A set with the minimum single-valued neutrosophic cardinality of $\gamma_{IDN}(\xi)$ is denoted as an γ_{IDN}-set or minimal SVNIDS.

Example 5: Consider $\xi = (A,B,C)$ to be denoted as an SVNIG as shown in **Fig. 4**. By routine calculations, the edges jk, kl, kn, and mn are single-valued neutrosophic incidence valid edges. The sets $D_1 = \{k,m\}$, $D_2 = \{k,n\}$, $D_3 = \{j,l,m\}$, $D_4 = \{j,l,n\}$ are SVNIDSs. By calculating the cardinalities, we obtain

$$|D_1| = \left| \frac{1+0.4+0.4-0.5}{2} + \frac{1+0.2+0.4-0.3}{2} \right| = 0.65 + 0.65 = 1.30,$$

$$|D_2| = \left| \frac{1+0.4+0.4-0.5}{2} + \frac{1+0.1+0.3-0.5}{2} \right| = 0.65 + 0.45 = 1.10,$$

$$|D_3| = \left| \frac{1+0.3+0.5-0.4}{2} + \frac{1+0.2+0.6-0.5}{2} + \frac{1+0.2+0.4-0.3}{2} \right| = 0.7 + 0.65 + 0.65 = 2.00,$$

$$|D_4| = \left| \frac{1+0.3+0.5-0.4}{2} + \frac{1+0.2+0.6-0.5}{2} + \frac{1+0.1+0.3-0.5}{2} \right| = 0.7 + 0.65 + 0.45 = 1.80.$$

Hence, $\gamma_{IDN}(\xi) = 1.10$ and $D_2 = \{k,n\}$ is a γ_{IDN}-set.

Theorem 1: In any SVNIG $\xi = (A,B,C)$, $\gamma_{IDN}(\xi) \leq P - \Delta_{IVN}$.

Proof: Let $x \in V$ in an SVNIG $\xi = (A,B,C)$. Assume that $dN_{svniv}(x) = \Delta_{IVN}$. Then, $V - N_{svniv}(x)$ is an SVNIDS of ξ, such that

$$\gamma_{IDN}(\xi) \leq |V - N_{svniv}(x)| = P - \Delta_{IVN}.$$

■

Note that D denotes an incidence DS of the SVNIG $\xi = (A,B,C)$ as well as $D_1 \subseteq V$ provided that $D_1 \supseteq D$. Thus, D_1 also denotes an incidence DS. Subsequently, any subset D_2 of D is not required to be an incidence DS. Therefore, we are inclined to consider an SVNIG's minimal incidence DSs.

Definition 16: An SVNIDS D of the SVNIG $\xi = (A,B,C)$ is denoted as a minimal SVNIDS provided that any proper subset of D is not included in an SVNIDS of ξ.

The upper SVNIDN Γ_{IDN} equals the maximum cardinality of the minimal SVNIDS in ξ. Obviously, γ_{IDN} denotes the minimum cardinality of the minimal SVNIDS in ξ.

Example 6: In Example 3, the sets

$D_1 = \{h, j, e\}$, $D_2 = \{g, j, e\}$,

$D_3 = \{h, j, f\}$, $D_4 = \{f, j, g\}$

resemble the minimal SVNIDSs. Thus, $\Gamma_{IDN} = 2$ and $\gamma_{IDN} = 1.7$.

Definition 17: The vertex's valid degree $x \in V$ in an SVNIG $\xi = (A,B,C)$ is expressed to denote the sum of the falsity membership, indeterminacy membership, and true membership degree of the single-valued neutrosophic incidence valid edges (SVNIVE) incident at vertex $x \in V$, which is expressed as $d_{svniv}(x)$. The maximum and minimum valid degree cardinalities of $\xi = (A,B,C)$ are denoted as Δ_{svniv} and δ_{svniv}, respectively.

Definition 18: Let the SVNIG $\xi = (A,B,C)$, a vertex $x \in V$ is denoted as an isolated vertex provided that $N_{svniv}(x) = \emptyset$. This implies that, for any $y \in V$, where $y \neq x$, xy is not a SVNIVE.

Example 7: As may be clearly observed from **Fig. 4**, j denotes an isolated vertex due to $N_{svniv}(j) = \emptyset$.

Theorem 2: In an SVNIG $\xi = (A,B,C)$ possess no isolated vertices. Provided that D denotes the minimal SVNIDS in ξ, $V - D$ thus resembles an SVNIDS.

Proof: Let D resemble any minimal SVNIDS of ξ. Meanwhile, the vertex $x \in D$ incidentally is not dominated by any vertex in $V - D$. Provided that ξ possesses no isolated vertices, x is incidentally dominated by at least one vertex in $D - \{x\}$. Thus, $D - \{x\}$ denotes an SVNIDS, which contradicts the minimality of D. Hence, any vertex in D is incidentally dominated by at least one vertex in $V - D$. Therefore, $V - D$ resembles an SVNIDS. ■

Corollary 1: For an SVNIG $\xi = (A,B,C)$ with no isolated vertex, we now have $\gamma_{IDN} \leq P/2$.

Proof: Provided that D denotes a minimal SVNIDS of ξ, so does $V - D$. Therefore, $P = |V| = |D| + |V - D|$. Hence, at least one of the sets D or $V - D$ has a cardinality $P/2$ or less. ■

4.2. Irredundant Sets in an SVNIG

Definition 19: Consider $\xi = (A,B,C)$ to be denoted as an SVNIG, $S \subseteq V$ as well as $x \in S$. The vertex y is known as a single-valued neutrosophic incidence valid private neighborhood (SVNIPN) of x into S provided that $N_{svniv}[x] \cap S = \{x\}$. In addition, we express the SVNIPN of x into S by $PN_{svniv}(x,S)$. This expresses

$$PN_{svniv}(x,S) = N_{svniv}[x] - \bigcup_{y \in S - \{x\}} N_{svniv}[y]$$
$$= N_{svniv}[x] - N_{svniv}[S - \{x\}].$$

Clearly, provided that $x \in PN_{svniv}(x,S)$, x is isolated in $\langle S \rangle$.

Definition 20: Let $\xi = (A,B,C)$ denote an SVNIG as well as $\emptyset \neq S \subseteq V$.

1. S is a single-valued neutrosophic incidence irredundant set (SVNIIRS) provided that for any $x \in S$, $PN_{svniv}(x,S) \neq \emptyset$.
2. S is considered a maximal SVNIIRS provided that, for any $x \in V - S$, the set $S \cup \{x\}$ is not an SVNIIRS, which explains that there exists at least one vertex $y \in S \cup \{x\}$ where it does not possess an SVNIPN.
3. The minimum cardinality among all maximal-SVNIIRSs is considered as a single-valued neutrosophic incidence irredundant number (SVNIIRN) and expressed by $ir_N(\xi)$.
4. The maximum cardinality among all maximal-SVNIIRSs is known as an upper SVNIIRN and expressed by $Ir_N(\xi)$. Hence, it may be easily noted that $ir_N(\xi) \subseteq Ir_N(\xi)$.

Theorem 3: Let SVNIG $\xi = (A,B,C)$ with vertex cardinality P and minimum cardinality single-valued neutrosophic incidence valid neighborhood (SVNIVN) degree δ_{IVN}. Then,

$Ir_N(\xi) \leq P - \delta_{IVN}$.

Proof: Let S denote an SVNIIRS in ξ with $x \in S$. Conclude that x resembles a valid neighborhood to k vertices in S. Here, given that the degree of x denotes at least δ_{IVN}, x must be valid neighborhood to at least $\delta_{IVN} - dN_k$ vertices in $V - S$ in which dN_k resembles the cardinality of k valid neighborhood vertices x in S.

Provided that $dN_k = 0$, $\delta_{IVN} \leq |V - S|$, i.e., $|S| \leq P - \delta_{IVN}$ as needed.

Provided that $dN_k > 0$, each valid neighborhood of x in S must possess a valid private neighborhood in $V - S$ and k must be distinct.

Therefore,

$$(\delta_{IVN} - dN_k) + dN_k \leq |V - S|,$$
$$\delta_{IVN} \leq |V - S|,$$

where

$$|S| \leq P - \delta_{IVN},$$
$$Ir_N(\xi) \leq P - \delta_{IVN}.$$

■

Theorem 4: An SVNIDS in an SVNIG $\xi = (A,B,C)$ is a minimal SVNIDS provided that it is an SVNIIRS.

Proof: Let S denote a minimal SVNIDS in ξ. Then, for any vertex $x \in S$, there exists a vertex $z \in V-(S-\{x\})$ that is not dominated by $S-\{x\}$. Thus, for any $x \in S$, $PN_{svniv}[x,S] \neq \emptyset$. Hence, any vertex $x \in S$ possesses at least one incidence valid private neighborhood. Therefore, S is both an SVNIIRS and an SVNIDS.

Therefore, provided that the set S is not a minimal SVNIDS, there exists an $x \in S$ given that $S-\{x\}$ is an SVNIDS. Because S is an SVNIIRS, $PN_{svniv}[x,S] \neq \emptyset$. Given that $z \in PN_{svniv}[x,S]$, z is not a valid neighborhood for any vertex in $S-\{x\}$. Provided that $S-\{x\}$ is not an SVNIDS, this leads to a contradiction. ■

Theorem 5: Every minimal SVNIDS in an SVNIG $\xi=(A,B,C)$ is a maximal SVNIIRS.

Proof: By Theorem 4, any minimal SVNIDS is an SVNIIRS. Thus, we need to show that S is a maximal SVNIIRS provided that S is not maximal. Thus, there exists a vertex $x \in V-S$ in which $S \cup \{x\}$ is an SVNIIRS. This shows that $PN_{svniv}[x,S\cup\{x\}] \neq \emptyset$. Thus, there exists at least one vertex y that is a valid private neighborhood of x into $S \cup \{x\}$. Ultimately, no vertex in S is a neighborhood to y. This contradicts S being an SVNIDS. ■

Corollary 2: For any SVNIG $\xi=(A,B,C)$,
$ir_N(\xi) \leq \gamma_{IDN}(\xi) \leq \Gamma_{IDN}(\xi) \leq Ir_N(\xi)$.

Proof: By Theorem 5, every minimal SVNIDS in $\xi=(A,B,C)$ is a maximal SVNIIRS in ξ. Thus, the result is straightforward. ■

4.3. Independent Sets in an SVNIG

Definition 21: Let $\xi=(A,B,C)$ denote an SVNIG. Then,

1. The vertices y and x are incidentally independent in ξ provided that the edge xy is not an SVNIVE.
2. The subset S of V is a single-valued neutrosophic incidence independent set (SVNIIDS) of ξ provided that any two vertices of S are incidentally independent. This shows that all of the vertices of $\langle S\rangle$ are isolated.
3. S is referred to as a maximal SVNIIDS in which for any $x \in V-S$ the set $S\cup\{x\}$ is not incidence-independent.
4. The maximum cardinality among all maximal SVNIIDS is referred to as a single-valued neutrosophic incidence-independent number (SVNIIDN) of ξ given by $\beta_{IDS}(\xi)$.
5. The minimum cardinality among all maximal SVNIIDS is a lower SVNIIDN of ξ given by $\alpha_{IDS}(\xi)$.

Theorem 6: Let $\xi=(A,B,C)$ be an SVNIG. Then, SVNIIDS denotes a maximal provided that it is an SVNIDS.

Proof: Let D be a maximal SVNIIDS of ξ. Then, D must be an SVNIDS of ξ. If not, there exists a vertex $x \in V-D$ that incidentally is not dominated by D, and thus $D\cup\{x\}$ denotes SVNIIDS of ξ, violating the maximality of D. Interchangeably, provided that D denotes independent SVNIDS, for any $x \notin D$, $D\cup\{x\}$ is not an SVNIIDS. Thus, D resembles a maximal SVNIIDS. ■

Theorem 7: Every maximal SVNIIDS of ξ denotes a minimal SVNIDS of ξ.

Proof: Proof. Let D be a maximal SVNIIDS of ξ. Then, by Theorem 6, D is an SVNIDS of ξ. Therefore, we must determine whether D is a minimal SVNIDS. If not, there exists at least one vertex $x \in D$ in which $D-\{x\}$ is an SVNIDS. Interchangeably, provided that $D-\{x\}$ dominates x, at least one vertex in $D-\{x\}$ must be SVNIVN of x, which contradicts D and resembles an SVNIIDS of ξ. Moreover, D must be a minimal SVNIDS. ■

Corollary 3: Each SVNIIDS of ξ is a minimal SVNIDS of ξ.

Proof: Let S be an SVNIIDS of ξ. Then, Theorem 6 resembles a maximal independent set and Theorem 7 resembles a minimal SVNIDS. ■

The converse of Corollary 3 may not be accurate in general, as shown in the example given below.

Corollary 4: For any SVNIG $\xi=(A,B,C)$, we have
$\gamma_{IDN}(\xi) \leq \alpha_{IDS}(\xi) \leq \beta_{IDS}(\xi) \leq \Gamma_{IDN}(\xi)$.

Proof: By Theorem 7, any maximal SVNIIDS D of ξ is a minimal SVNIDS of ξ. Then, the result is straightforward. ■

By Corollaries 2 and 4, we obtain the following result.

Theorem 8: For any SVNIG $\xi=(A,B,C)$, $ir_N(\xi) \leq \gamma_{IDN}(\xi) \leq \alpha_{IDS}(\xi) \leq \beta_{IDS}(\xi) \leq \Gamma_{IDN}(\xi) \leq Ir_N(\xi)$.

5. An Application of SVNIGs to Determining Optimal Locations of CVACs

The COVID-19 pandemic is a global outbreak of severe acute respiratory syndrome (SARS-CoV-2). Vaccination is one of the most promising techniques to mitigate the impacts of the pandemic. Since the beginning of the outbreak, many initiatives have been conducted worldwide to create COVID-19 vaccines. The Special Committee for Ensuring Access to COVID-19 Vaccine Supply (JKJAV), co-chaired by Malaysian Ministers of Health and Science, Technology, and Innovation was formed to assure the country's immediate access to COVID-19 vaccine supplies. The government's objective is to guarantee as many Malaysians as possible attain vaccination to minimize the mortality and morbidity of the disease as quickly as possible. Universities, community halls, convention centers, stadiums, and other suitable venues will be established as temporary vaccine administration centers depending on demand. However, the issue of where to locate these temporary vaccination administration centers is very important. To reduce costs, the government must choose the smallest number of cities that are the most appealing so that the remaining cities are connected to at most one city with available vaccines by a usable traveling route for better accessibility.

Therefore, in this section, we attempted to identify the most suitable cities to construct CVACs using SVNIGs

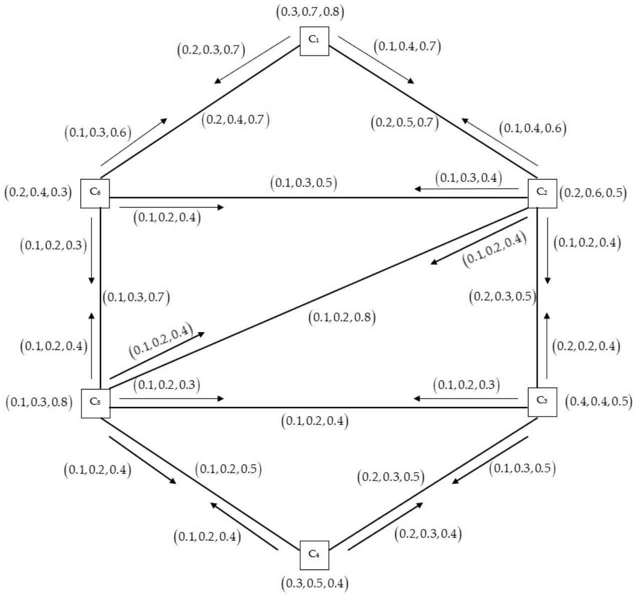

Fig. 5. Pictorial diagram of the application of SVNIG.

and DSs to save both expense and time. Moreover, the cities have diverse circumstances in regards to the CVAC location's development, and differ considerably in terms of their highways in relation to the safety, quality, distance, and the presence of checkpoints and traffic at various levels. The cities in the two regions are modeled as the set of vertices (V), and the routes connecting cities are represented by the set of edges (E). The city entry and exit points (the ramp between the road and the city) are modeled by the set of incidences (I) in this graph. **Fig. 5** illustrates this application for the problem of locating CVACs.

We considered six cities designated as C_1, C_2, C_3, C_4, C_5, and C_6. In this notation, a vertex $C_2(0.2,0.6,0.5)$ has 20% of the facilities required to set up a CVAC. A value of 50% indicates a lack of necessary equipment, while 60% indicates a correspondingly more notable insufficiency. The edge $C_2C_3(0.2,0.3,0.5)$ shows that 20% of the route to the CVAC is not disrupted by any traffic. Still, unfortunately, 50% of the route between these two points is congested with vehicles, especially during the rush hours, while 30% of the route is not disrupted by any traffic. The DSs for **Fig. 5** were calculated as follows:

$$\begin{array}{ll} D_1=\{C_2,C_3,C_4\}, & D_2=\{C_2,C_3,C_5\}, \\ D_3=\{C_1,C_2,C_4\}, & D_4=\{C_1,C_2,C_3,C_4\}, \\ D_5=\{C_2,C_3,C_4,C_5\}, & D_6=\{C_2,C_3,C_4,C_6\}, \\ D_7=\{C_1,C_2,C_4,C_6\}, & D_8=\{C_1,C_3,C_5,C_6\}, \\ D_9=\{C_2,C_4,C_5,C_6\}, & D_{10}=\{C_1,C_3,C_4,C_5\}. \end{array}$$

After calculating the cardinality of $D_1,\ldots,D_{10}$, we obtain

$$\begin{array}{ll} |D_1|=2, & |D_2|=1.6, \\ |D_3|=1.95, & |D_4|=2.6, \\ |D_5|=2.3, & |D_6|=2.65, \\ |D_7|=2.6, & |D_8|=2.2, \\ |D_9|=2.3, & |D_{10}|=2.25. \end{array}$$

Because D_2 has a minimum scalar cardinality among SVNIDSs, we conclude that C_2, C_3, and C_5 can be selected as the preferred cities in which to construct the CVACs. Considering the facilities and equipment in all the cities, these locations are most well supplied and equipped. Thus, we conclude that the government must allocate sufficient funds to these cities and provide more health officers and volunteers to established CVACs in order for our country to attain greater immunity as soon as possible.

6. Conclusion and Future Research

In graph theory, the concept of domination is essential from both theoretical and practical perspectives. Domination in incidence graphs has been utilized to formulate and solve a variety of issues in science and technology, such as in combinatorial analysis, artificial intelligence, computer networks, etc. NSs are an extension of IFSs and FS notion. In comparison to classical and fuzzy models, NS models provide higher compatibility, flexibility, and precision.

In this study, we integrated the concept of domination with the idea of SVNIGs and also covered several important graph-theoretic concepts. Each of these concepts wasexplored with appropriate instances. We also introduced the concepts of single-valued neutrosophic incidence valid edges and a definition of cardinality in SVNIGs. In addition, we explored many key results relating to these dominations, such as irredundant and independent sets and DSs in SVNIGs employing valid edges. Lastly, to demonstrate a real-world application of domination in an SVNIG model, we suggested an exemplary scenario relating to the problem of selecting locations for CVACs.

Further studies can be carried out to determine the domination in SVNIGs using strong or effective pairs. Furthermore, we plan to continue our study of various domination parameters in NSs and identify the bounds of domination parameters.

Conflicts of Interest

The authors declare no conflict of interest.

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Name:
Siti Nurul Fitriah Mohamad

Affiliation:
College of Computing, Informatics and Mathematics, Universiti Teknologi MARA

Address:
Machang, Kelantan 18500, Malaysia

Brief Biographical History:
2010 Received B.Sc. degree in Mathematical Science from Universiti Sains Malaysia (USM)
2011 Received M.Sc. degree in Mathematical Science from USM

Main Works:
- Neutrosophic graph theory.

Name:
Roslan Hasni

ORCID:
0000-0003-3695-2145

Affiliation:
Special Interest Group on Modeling and Data Analytics, Faculty of Ocean Engineering Technology and Informatics, Universiti Malaysia Terengganu

Address:
Kuala Nerus, Terengganu 21300, Malaysia

Brief Biographical History:
1998 Received B.Sc. degree in Education (Mathematics) from Universiti Sains Malaysia
2000 Received M.Sc. degree in Mathematics from Universiti Kebangsaan Malaysia
2005 Received Ph.D. degree in Mathematics from Universiti Putra Malaysia
2012- Associates Professor, Universiti Malaysia Terengganu

Main Works:
- Graph theory.

Name:
Florentin Smarandache

ORCID:
0000-0002-5560-5926

Affiliation:
Mathematics & Science Department, The University of New Mexico

Address:
705 Gurley Avenue, Gallup, New Mexico 87301, USA

Brief Biographical History:
2008- Professor of Mathematics, The University of New Mexico
2009 Postdoctoral Researcher, Air Force Research Laboratory
2010 Postdoctoral Researcher, École nationale supérieure de techniques avancées Bretagne
2013-2014 Postdoctoral Researcher, Okayama University of Science
2014 Postdoctoral Researcher, Guangdong University of Technology

Main Works:
- Neutrosophic set.