FLORENTIN SMARANDACHE
Non-Geometry

NON-GEOMETRY

It's a lot easier to deny Euclid’s five postulates than Hilbert’s twenty thorough axioms.

1. It is not always possible to draw a line from an arbitrary point to another arbitrary point.
   For example: this axiom can be denied only if the model’s space has at least a discontinuity point; (in our bellow model $MD$, one takes an isolated point $I$ in between $f_1$ and $f_2$, the only one which will not verify the axiom).

2. It is not always possible to extend by continuity a finite line to an infinite line.
   For example: consider the bellow Model, and the segment $AB$, the both $A$ and $B$ lie on $f_1$. $A$ in between $P$ and $N$, while $B$ on the left side of $N$, one can not at all extend $AB$ either beyond $A$ or beyond $B$, because the resulted curve, noted say $A' - A - B - B'$, would not be a geodesic (i.e. line in our Model) anymore.
   If $A$ and $B$ lie in $\delta_1 - f_1$, both of them closer to $f_1$, $A$ in the left side of $P$, while $B$ in the right side of $P$, then the segment $AB$, which is in fact $A - P - B$, can be extended beyond $A$ and also beyond $B$ only up to $f_1$ (therefore one gets a finite line too, $A' - P - B - B'$, where $A', B'$ are the intersections of $PA, PB$ respectively with $f_1$).
   If $A, B$ lie in $\delta_1 - f_1$, far enough from $f_1$ and $P$, such that $AB$ is parallel to $f_1$, then $AB$ verifies this postulate.

3. It is not always possible to draw a circle from an arbitrary point and of an arbitrary interval.
   For example: same as for the first axiom; the isolated point $I$, and a very small interval not reaching $f_1$, neither $f_2$, will deny this axiom.

4. Not all the right angles are congruent. (See example of the Anti-Geometry, explained bellow.)

5. If a line, cutting two other lines, forms the interior angles of the same side of it strictly less than two right angles, then not always the two lines extended towards infinite cut each other in the side where the angles are strictly less than two right angles.
   For example: let $h_1, h_2, i$ be three lines in $\delta_1 - \delta_2$, where $h_1$ Intersects $f_1$ in $A$, and $h_2$ intersects $f_1$ in $B$, with $A, B, P$ different each other, such that $h_1$ and $h_2$ do not intersect, but $i$ cuts $h_1$ and $h_2$ and forms the interior angles of one of its side (towards $f_1$) strictly less than two right angles;
the assumption of the fifth postulate is fulfilled, but the consequence does not hold, because \( h_1 \) and \( h_2 \) do not cut each other (they may not be extended beyond \( A \) and \( B \) respectively, because the lines would not be geodesics anymore).

**Question 29**

Find a more convincing model for this non-geometry.