Open Questions about Concatenated Primes and Metasequences

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Abstract.
We define a metasequence as a sequence constructed with the terms of other given sequence(s).
In this short note we present some open questions on concatenated primes involved in metasequences.

First Class of Concatenated Sequences.

1) Let \( a_1, a_2, \ldots, a_{k-1}, a_k \) be given \( k \geq 1 \) digits in the numeration base \( b \).
   a) There exists a prime number \( P \) of the concatenated form:

   \[
P = *\ldots*a_1*\ldots*a_2*\ldots*\ldots*a_{k-1}*\ldots*a_k*
   \]

   where the stars “*...*” represent various (from none to any finite positive integer) numbers of digits in base \( b \).

   Of course, if \( a_k \) is the last digit then \( a_k \) should belong to the set \{1, 3, 7, 9\} in base 10. Similar restriction for the last number’s digit \( a_k \) in other base \( b \).

   b) Are there infinitely many such primes?
   c) What about considering fixed positions for the given digits: i.e. each given \( a_i \) on a given position \( n_i \) ?
   d) As a consequence, for any group of given digits \( a_1, a_2, \ldots, a_{k-1}, a_k \) do we have finitely or infinitely many primes starting with this group of digits (i.e. in the following concatenated form):

   \[
a_1a_2...a_{k-1}a_k*\ldots*
   \]

   ?

   e) As a consequence, for any group of given digits \( a_1, a_2, \ldots, a_{k-1}, a_k \) do we have finitely or infinitely many primes ending with this group of digits (i.e. in the following concatenated form):

   \[
*\ldots*a_1a_2...a_{k-1}a_k
   \]
f) As a consequence, for any group of given digits \(a_1, a_2, \ldots, a_{k-1}, a_k\) and any given digits \(b_1, b_2, \ldots, b_{j-1}, b_j\) do we have finitely or infinitely many primes beginning with the group of digits \(a_1, a_2, \ldots, a_{k-1}, a_k\) and ending with the group of digits \(b_1, b_2, \ldots, b_{j-1}, b_j\) (i.e. in the following concatenated form):

\[
a_1a_2\ldots a_{k-1}a_k \ast \cdots \ast b_1b_2\ldots b_{j-1}b_j
\]

(of course considering the primality restriction on the last digit \(b_j\))?

g) As a consequence, for any group of given digits \(a_1, a_2, \ldots, a_{k-1}, a_k\) do we have finitely or infinitely many primes having inside of their concatenated form this group of digits (i.e. in the following concatenated form):

\[
\ast \cdots \ast a_1a_2\ldots a_{k-1}a_k \ast \ast \ast
\]

h) As a consequence, for any groups of given digits \(a_1, a_2, \ldots, a_{k-1}, a_k\) and \(b_1, b_2, \ldots, b_{j-1}, b_j\) and \(c_1, c_2, \ldots, c_{i-1}, c_i\) do we have finitely or infinitely many primes beginning with the group of digits \(a_1, a_2, \ldots, a_{k-1}, a_k\), ending with the group of digits \(b_1, b_2, \ldots, b_{j-1}, b_j\), and having inside the group of digits \(c_1, c_2, \ldots, c_{i-1}, c_i\) (i.e. in the following concatenated form):

\[
a_1a_2\ldots a_{k-1}a_k \ast \cdots \ast c_1c_2\ldots c_{i-1}c_i \ast \ast \ast b_1b_2\ldots b_{j-1}b_j
\]

(of course considering the primality restriction on the last digit \(b_j\))?

i) What general condition has a sequence \(s_1, s_2, \ldots, s_n\) to satisfy in order for the concatenated metasequence

\[
S_1S_2\ldots S_n
\]

for \(n = 1, 2, \ldots\) to contain infinitely many primes?

**Second Class of Metasequences.**

2) Let’s note the sequence of prime numbers by \(p_1 = 2, p_2 = 3, p_3 = 5, \ldots, p_n\) the \(n^{\text{th}}\) prime number, for any natural number \(n\).

a) Does the metasequence

\[
p_1p_2\ldots p_n + 1
\]
for $n = 1, 2, \ldots$ contains finitely or infinitely many primes?

b) What about the metasequence:

$$p_1 p_2 \cdots p_n - 1$$

?

c) What general condition has a sequence $s_1, s_2, \ldots, s_n, \ldots$ to satisfy in order for the metasequence

$$s_1 s_2 \cdots s_n \pm 1$$

for $n = 1, 2, \ldots$ to contain infinitely many primes?

Reference:

F. Smarandache, Sequences of Numbers Involved in Unsolved Problems, 139 p., HeXis, 2006.