Applications of Extencs to 2D-Space and 3D-Space

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Abstract. In this article one proposes several numerical examples for applying the extension set to 2D- and 3D-spaces. While rectangular and prism geometrical figures can easily be decomposed from 2D and 3D into 1D linear problems, similarly for the circle and the sphere, it is not possible in general to do the same for other geometrical figures.

Key words. Extencs, extension engineering, contradictory problems, computational and artificial intelligence

I. INTRODUCTION

Extencs has been used since 1983 by Cai Wen [1,2,4,5] and many other Chinese scholars [3, 6,7] in solving contradictory problems. Its goal is to solve contradictory problems and also nonconventional, nontraditional ideas in many fields. Extencs is at the confluence of three disciplines: philosophy, mathematics, and engineering. A contradictory problem is converted by a transformation function into a non-contradictory one. The functions of transformation are: extension, decomposition, combination, etc. Extencs has many practical applications in Management, Decision-Making, Strategic Planning, Methodology, Data Mining, Artificial Intelligence, Information Systems, Control Theory, etc.

Extencs is based on matter-element, affair-element, and relation-element. The distance between a number and a set, and the degree of dependence of a point with respect to a set were defined for the one-dimensional space, and later for higher dimensional spaces. We present below several examples in 2D and 3D spaces.

2. APPLICATION 1.

We have a factory piece whose desired 2D-dimensions should be 20 cm × 30 cm, and acceptable 2D-dimensions 22 cm × 34 cm. We define the extension 2D-distance, and then we compute the extension 2D-dependent function. Let’s do an extension diagram from the figure 1.

We have a desirable factory piece A’B’C’D’ and an acceptable factory piece ABCD. The optimal point for both of them is O(17,11).

a) The region determined by the rays OA and OD.

The extension 2D-distance \( \rho \) between a point P and a set is the distance from P to the closest frontier of the set, distance measured on the line OP.

Diagram 1.

Whence

\[
\rho(P, A'B'C'D') = -|PP_1| \quad (1)
\]

and

\[
\rho(P, ABCD) = -|PP_2| \quad (2)
\]

The extension 2D-dependent function \( k \) of a point P which represents the dependent of the point of the nest of the two sets is:

\[
k(P) = \frac{\rho(P, \text{big set}) - \rho(P, \text{small set})}{\rho(P, \text{big set}) + \rho(P, \text{small set})} = \frac{|PP_1|}{|PP_1| + |PP_2|} \quad (3)
\]

In other words, the extension 2D-dependent function \( k \) of a point P is the 2D-extension distance between the point and the closest frontier of the larger set, divided by the 2D-extension distance between the frontiers of the two nested sets; all these 2D-extension distances are taken along the line OP.

In our application one has:

\[
k(P) = \frac{|PP_2|}{|PP_1|} \quad (4)
\]

since P is inside of the larger set. If P was outside of the larger set, then \( k(P) \) would be negative.

Let’s consider the coordinates of \( P(x_0,y_0) \), where P is between the rays OA and OD in order to make sure OP intersects the line segments AD and A’D’ which are closest frontiers of the rectangles ABCD and respectively A’B’C’D’. {The problem would be similar if P was in between the rays OB and OC.}
Hence \( y_0 \in (11, \infty) \) but such \( y_0 \) that remains in between the rays \( OA \) and \( OD \).

Let’s find the coordinates of \( P_1 \). In analytical geometry the equation of line \( OP \) passing through two points, \( O(17,11) \) and \( P(x_0,y_0) \), is:

\[
y - 11 = \frac{y_0 - 11}{x_0 - 17} (x - 17).
\]

Since the \( y \)-coordinate of \( P_1 \) is 21, we replace \( y = 21 \) in the above equation and we get the \( x \)-coordinate of \( P_1 \).

Whence one has:

\[
P_1\left(\frac{10x_0 + 17y_0 - 357}{y_0 - 11}, 21\right).
\]

Let’s find the coordinates of \( P_2 \). The \( y \)-coordinate of \( P_2 \) is 22. Replace \( y = 22 \) in equation (2) and solve for the \( x \)-coordinate of \( P_2 \).

One gets

\[
P_2\left(\frac{11x_0 + 17y_0 - 374}{y_0 - 11}, 22\right).
\]

The classical distance in 2D-space between two points \( M(m_1, m_2), N(n_1, n_2) \) is

\[
d(M, N) = \sqrt{(m_1 - n_1)^2 + (m_2 - n_2)^2}.
\]

We compute the classical 2D-distances \( d(P, P_2) \) and \( d(P_1, P_2) \).

\[
k(P) = \pm \frac{d(P, P_1)}{d(P, P_2)} = \pm \sqrt{\frac{11x_0 + 17y_0 - 374}{y_0 - 11} - \frac{11x_1 + 17y_1 - 374}{y_1 - 11} + \frac{22x_0 - 22y_0 - 374}{y_0 - 11} + \left(\frac{22y_0 - 22}{y_0 - 11}\right)^2}.
\]

\[
= \pm \sqrt{\frac{22y_0 + 22}{y_0 - 11} + \left(\frac{22y_0 - 34}{y_0 - 11}\right)^2}.
\]

\[
= \pm |y_0 - 22| \quad \text{for } y_0 \in [11, 22],
\]

\[
= 22 - y_0 \quad \text{for } y_0 > 22.
\]

\[
(7)
\]

and \( P \) in between the rays \( OA \) and \( OD \).

Since the extension 2D-dependent function \( k(x_0, y_0) = 22 - y_0 \) for \( y_0 > 11 \), does not depend on \( x_0 \) for the region between rays \( OA \) and \( OD \), one has classes of points lying on horizontal lines parallel to \( A'D' \) (see the green line segments on Diagram 1) whose extension 2D-dependent function value is the same. For example, the green horizontal line segment passing thought \( P \) is the class of points having the same extension 2D-dependent function value as point \( P \).

b) The region determined by the rays \( OC \) and \( OD \).

\( \{ \text{Similar result would obtain if one gets the opposite} \}

region determined by the rays \( OA \) and \( OB \} \)

If one takes another region determined by the rays \( OC \) and \( OD \) and a point \( Q(x_1, y_1) \) in between one gets

\[
k(Q) = k(x_1, y_1) = \pm \frac{|QQ_1|}{|QQ_2|}.
\]

By a similar method we find the Cartesian coordinates of the points \( Q_1 \) and \( Q_2 \).

In analytical geometry the equation of line \( OQ \) passing through two points, \( O(17,11) \) and \( Q(x_1, y_1) \), is:

\[
y - 11 = \frac{y_1 - 11}{x_1 - 17} (x - 17).
\]

Since the \( x \)-coordinate of \( Q_1 \) is 32, we replace \( x = 32 \) in the above equation and we get the \( y \)-coordinate of \( P_1 \).

Whence one has

\[
Q_1(32, \frac{11x_1 + 15y_1 - 352}{x_1 - 17}).
\]

Let’s find the coordinates of \( Q_2 \). The \( x \)-coordinate of \( P_2 \) is 34. Replace \( x = 22 \) in equation (3) and solve for the \( y \)-coordinate of \( Q_2 \). One gets

\[
Q_2(34, \frac{11x_1 + 17y_1 - 374}{x_1 - 17}).
\]

We compute the classical 2D-distances \( d(Q, Q_2) \) and \( d(Q_1, Q_2) \).

\[
k(Q) = \pm \frac{|QQ_1|}{|QQ_2|} = \pm \sqrt{\left(\frac{34 - x_1}{2}\right)^2 + \left(\frac{11x_1 + 17y_1 - 374}{x_1 - 17} - y_1\right)^2}.
\]

\[
= \pm \sqrt{\left(\frac{x_1 - 34}{2}\right)^2 + \left(\frac{11x_1 + 17y_1 - 374}{x_1 - 17} - 11x_1 + 15y_1 - 352\right)^2}.
\]

\[
= \pm \sqrt{\left(\frac{x_1 - 34}{2}\right)^2 + \left(\frac{11x_1 - 15y_1}{x_1 - 17}\right)^2}.
\]

and \( Q \) in between the rays \( OC \) and \( OD \).

Since the extension 2D-dependent function \( k(x_1, y_1) = \frac{34 - x_1}{2} \), for \( x_1 > 17 \), does not depend on \( y_1 \) for the region between rays \( OC \) and \( OD \), one has classes of points lying on vertical lines parallel to \( C'D' \) (see the red line segments on Diagram 1) whose extension 2D-dependent function value is the same. For example, the blue vertical line segment passing thought \( Q \) is the class of points having the same extension 2D-dependent function value as point \( Q \).

2. SPLITTING AN EXTENSION 2D-PROBLEM INTO TWO 1D-PROBLEMS.

Remarkably, for rectangular shapes one can decompose a 2D-problem into two 1D-problems. Yet, for other geometrical figures it is not possible. The more irregular geometrical
figure, the less chance to decompose a 2D-problem into 1D-problems. In our case, we separately consider the factory piece’s width and length.

1) The width of a factory piece is desirable to be 20 cm and acceptable up to 22 cm.
2) The length of a factory piece is desirable to be 30 cm and acceptable up to 34 cm.

In the first 1D-problem one makes the diagram:

One computes, using Prof. Cai Wen’s extension 1D-dependent function:
\[ k(y_0) = \frac{22 - y_0}{\frac{22 - y_0 - (y_0 - 11)}{2}} \]

If \( y_0 > 11 \) as in our 2D-space problem, then \( k(y_0) = 22 - y_0 \)
which is consistent with what we got in the 2D case.

In the second 1D-problem one makes the diagram:

One computes, using Prof. Cai Wen’s extension 1D-dependent function:
\[ k(x_0) = \frac{34 - x_0}{\frac{34 - x_0 - (x_0 - 17)}{2}} \]

If \( x_0 > 17 \) as in our 2D-space problem, then
\[ k(x_0) = 34 - x_0 \]
which is consistent with what we got in the 2D-case.

Therefore, a 2D-extension problem involving rectangles is equivalent to two 1D-extension problems. Certainly this equivalence is not valid any longer if instead of rectangles we have more irregular geometrical figures representing factory pieces.
Similarly will be possible for splitting a 3D-application for prisms into three 1D-applications, or into one 2D-application and one 1D-application.

3. REGION CRITICAL ZONE.

Critical Zone is the region of points where the degree of dependence of a point \( P \) with respect to a nest of two intervals \( k(P) \in (-1, 0) \).

In the above figure, it is all area between the rectangles ABCD and \( A_1B_1C_1D_1 \).

\( A_1B_1C_1D_1 \) was constructed by drawing parallels to the sides of the rectangle ABCD, such that:
- The distance between the parallel lines \( A'D' \) and \( AD \), be the same with the distance between the parallel lines \( AD \) and \( A_1D_1 \);
- The distance between the parallel lines \( A'B' \) and \( AB \), be the same with the distance between the parallel lines \( AB \) and \( A_1B_1 \);
- The distance between the parallel lines \( B'C' \) and \( BC \), be the same with the distance between the parallel lines \( BC \) and \( B_1C_1 \);
- The distance between the parallel lines \( C'D' \) and \( CD \), be the same with the distance between the parallel lines \( CD \) and \( C_1D_1 \).

One then extend the construction of a net of included rectangles \( A_1B_1C_1D_1 \subset A_{i+1}B_{i+1}C_{i+1}D_{i+1} \) and for the points \( P_{i+1} \) lying on surface in between the rectangles \( A_1B_1C_1D_1 \) and \( A_{i+1}B_{i+1}C_{i+1}D_{i+1} \) the dependent function \( k(P_{i+1}) \in (-1, -1) \).

4. APPLICATION IN THE 3D-SPACE.

A factory piece has the desirable dimensions 20x30x7 but the acceptable factory piece can be 22x34x10 (in centimeters).

The red prism is the desirable form, and the green prism is the acceptable form.

We consider a Cartesian system XYZ and the vertexes of these two prisms are:
\[ A(0, 22, 0), \ B(0, 0, 0), \ C(34, 0, 0), \ D(34, 22, 0), \ E(0, 22, 10), \ F(0, 0, 10), \ G(34, 0, 10), \ H(34, 22, 10); \]
\[ A'(2, 21, 3), \ B'(2, 1, 3), \ C'(32, 1, 3), \ D'(32, 21, 3), \ E'(2, 21, 7), \ F'(2, 1, 7), \ G'(32, 1, 7), \ H'(32, 21, 7). \]

\( O(17, 11, 5); \ P(x_0, y_0, z_0), \ P'(x_1, y_1, 7), \ P''(x_2, y_2, 10); \)
Q(17,11,z_o), Q'(17,11,7), Q”(17,11,10). (15)
The following triangles are similar: △QOP, △Q'O'P',
△Q''OP''.
Using similarity of triangles, Thales Theorem, and
proportionalizations we get that:
\[ \frac{|PP'|}{|P'P''|} = \frac{|QQ''|}{|Q'Q''|} \]
which is equivalent to the equality of dependent function
values
\[ \text{of } k(P) = k(Q). \]
since
\[ k(P) = \pm \frac{\rho(P,ABCD\text{EFGH})}{\rho(P,\text{A'B'C'D'E'F'G'H'})} = \pm \frac{|PP'|}{|PP''|} = \pm \frac{|QQ'|}{|QQ''|} \] (16)
and similarly:
\[ k(Q) = \pm \frac{\rho(Q,\text{ABCD\text{EFGH})}}{\rho(Q,\text{A'B'C'D'E'F'G'H'})} = \pm \frac{|QQ'|}{|QQ''|} = \pm \frac{|Q'O'|}{|Q'O''|} \] (17)
Therefore, the plane which passes through the point P and is
parallel with the planes EFGH and E’F’G’H’ (limited by the
lines OE’, OF’, OG’, OH’) is the locus of points having the
same dependent function value.
\[ k(P) = \frac{z_o - 10}{3} \quad \text{for } z_o > 5 \quad \text{and point } P \text{ inside the reversed}
\text{pyramid } O\text{EFGH}. \]

5. CRITICAL ZONE.
The Critical Zone, whose dependent function of each point in
this zone belongs to (-1, 0), will be a larger prism
\( A_iB_iC_iD_iE_iF_iG_iH_i \) which envelops the prism
\( ABCDEFGH \) at the same distance from each face as it was between
the prisms \( A'B'C'D'E'F'G'H' \) and \( ABCDEFGH \). Therefore, the
distance between faces \( A'B'C'D' \) and \( ABCD \) is the same as
the distance between faces \( ABCD \) and \( A_iB_iC_iD_i \); and the
faces \( A'B'C'D' \) and \( ABCD \) and \( A_iB_iC_iD_i \) are parallel.
Similarly for all six faces of the prism \( A_iB_iC_iD_iE_iF_iG_iH_i \): the
distance between faces \( A'E'H'D' \) and \( AEHD \) is the same as
the distance between faces \( AEHD \) and \( A_iE_iH_iD_i \); and the
faces \( A'E'H'D' \) and \( AEHD \) and \( A_iE_iH_iD_i \) are parallel, etc.

One can construct a net of such prisms: \( A_{i_{1}}B_{i_{1}}C_{i_{1}}D_{i_{1}}E_{i_{1}}F_{i_{1}}G_{i_{1}}H_{i_{1}} \supseteq A_iB_iC_iD_iE_iF_iG_iH_i \) where
the value of the dependent function for the points which
belong to
\( \text{Int}(A_{i_{1}}B_{i_{1}}C_{i_{1}}D_{i_{1}}E_{i_{1}}F_{i_{1}}G_{i_{1}}H_{i_{1}} - A_iB_iC_iD_iE_iF_iG_iH_i) \) is in the
interval \( (-i-1, -i) \), while for the points lying on the
\( \text{Fr}(A_{i_{1}}B_{i_{1}}C_{i_{1}}D_{i_{1}}E_{i_{1}}F_{i_{1}}G_{i_{1}}H_{i_{1}}) \) the dependent function
is \( -i-1 \). One considers \( ABCDEFGH \) as \( A_0B_0C_0D_0E_0F_0G_0H_0 \) and
\( A'B'C'D'E'F'G'H' \) as \( A_jB_jC_jD_jE_jF_jG_jH_j \) for the rule to work for all included prisms.

6. SPLITTING A 3D-PROBLEM INTO THREE 1D-
PROBLEM.
Similarly to the previous 2D-problem, we separately consider
the factory piece’s width, length, and height.
1) The width of a factory piece is desirable to be 20 cm
and acceptable up to 22 cm.
2) And the length of a factory piece is desirable to be 30
and acceptable up to 34 cm.
3) And the height of a piece factory is desirable to be 7
and acceptable 10 cm.

In the first 1D-problem one makes the diagram:

\[ \text{Diagram 5.} \]
One computes, using Prof. Cai Wen’s extention 1D-dependent
function:
\[ k(y_0) = 11- | y_0 - 11| \] (18)
In the second 1D-problem one makes the diagram:

\[ \text{Diagram 6.} \]

In the third 1D-problem one makes the diagram:

\[ \text{Diagram 7.} \]
One computes, using Prof. Cai Wen’s extention 1D-dependent
function:
\[ k(z_0) = \frac{z_0 - 10}{3} \] (19)

7. SPLITTING A 3D-PROBLEM INTO A 2D-
PROBLEM AND A 1D-PROBLEM.
Similarly to the previous 2D-problem, we separately consider
the factory piece’s width, length, and height.

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1) The factory 2D-piece is desirable to be 20x30 cm and acceptable up to 22x34 cm.
2) And the height of a piece factory is desirable to be 7 cm and acceptable 10 cm.

8. A 2D-PROBLEM WHICH IS SPLIT INTO ONLY ONE 1D-PROBLEM.

Assume the desirable circular factory piece radius is 6 cm and acceptable is 8 cm.

Diagram 8.

It is equivalent to a 1D-problem which has the diagram:

Diagram 9.

One computes, using Prof. Cai Wen’s extension 1D-dependent function:

\[ k(x_0) = \frac{x_0}{2} \] (20)

9. A 2D-PROBLEM WHICH CANNOT BE SPLIT INTO 1D-PROBLEMS.

The Critical Zone is between the blue triangle \( A'B'C' \) and the black dotted triangle \( A''B''C'' \). Points lying on lines parallel to the red triangle’s sides have the same dependence function value (for example the points lying on the orange line segment).

Diagram 10.

7. CONCLUSION

In this paper we presented 2D-geometrical figures, such as two nested rectangles, two nested circles, and two nested triangles with no common ending points, and a 3D-geometrical figure, such as two nested prisms with no common ending points and we compute the dependent function values for a point with respect to these nested figures.

REFERENCES