In this article, we emphasize the radical axis of the Lemoine Circles. For the start, let us remind:

**Theorem 1.**

The parallels taken through the simmedian center $K$ of a triangle to the sides of the triangle determine on them six concyclic points (The First Lemoine Circle).

**Theorem 2.**

The antiparallels taken through the simmedian center of a triangle to the sides of a triangle determine on them six concyclic points (The Second Lemoine Circle).

**Remark 1.**

If $ABC$ is a scalene triangle and $K$ is its simmedian center, then $L$, the center of the First Lemoine Circle, is the middle of the segment $[OK]$, where $O$ is the center of the circumscribed circle, and the center of the Second Lemoine Circle is $K$. It follows that the radical axis of Lemoine circles is perpendicular on the line of the centers $LK$, therefore on the line $OK$.

**Proposition 1.**

The radical axis of Lemoine Circles is perpendicular on the line $OK$ raised in the simmedian center $K$. 

**Radical Axis of Lemoine Circles**

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Proof.

Let $A_1A_2$ be the antiparallel to $BC$ taken through $K$, then $KA_1$ is the radius $R_{L_2}$ of the Second Lemoine Circle; we have:

$$R_{L_2} = \frac{abc}{a^2+b^2+c^2}.$$  

![Figure 1](image)

Let $A_1'A_2'$ be the Lemoine parallel taken to $BC$; we evaluate the power of $K$ towards the First Lemoine Circle. We have:

$$\overline{KA_1'} \cdot \overline{KA_2'} = LK^2 - R_{L_1}^2. \quad (1)$$

Let $S$ be the simmedian leg from $A$; it follows that:

$$\frac{KA_1'}{BS} = \frac{AK}{AS} - \frac{KA_2'}{SC}.$$  

We obtain:

$$KA_1' = BS \cdot \frac{AK}{AS} \quad \text{and} \quad KA_2' = SC \cdot \frac{AK}{AS},$$

but $\frac{BS}{SC} = \frac{c^2}{b^2}$ and $\frac{AK}{AS} = \frac{b^2+c^2}{a^2+b^2+c^2}$.  

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Therefore:

\[
\overrightarrow{KA_1'} \cdot \overrightarrow{KA_2'} = -BS \cdot SC \cdot \left(\frac{AK}{AS}\right)^2 = \frac{-a^2b^2c^2}{(b^2+c^2)^2} \cdot \frac{(b^2+c^2)^2}{(a^2+b^2+c^2)^2} = -R_{L_2}^2. \tag{2}
\]

We draw the perpendicular in \(K\) on the line \(LK\) and denote by \(P\) and \(Q\) its intersection to the First Lemoine Circle; we have \(\overrightarrow{KP} \cdot \overrightarrow{KQ} = -R_{L_2}^2;\) by the other hand, \(KP = KQ\) (\(PQ\) is a chord which is perpendicular to the diameter passing through \(K\)).

It follows that \(KP = KQ = R_{L_2}\), so \(P\) and \(Q\) are situated on the Second Lemoine Circle.

Because \(PQ\) is a chord which is common to the Lemoine Circles, it follows that \(PQ\) is the radical axis.

**Comment 1.**

After equalizing relations (1) and (2) or by the Pythagorean theorem in the triangle \(PKL\), we can calculate \(R_{L_1}\). It is known that:

\[
OK^2 = R^2 - \frac{3a^2b^2c^2}{(a^2+b^2+c^2)^2},
\]

and since \(LK = \frac{1}{2} OK\), we find that:

\[
R_{L_1}^2 = \frac{1}{4} \left[ R^2 + \frac{a^2b^2c^2}{(a^2+b^2+c^2)^2} \right].
\]

**Remark 2.**

The proven *Proposition* regarding the radical axis of the Lemoine Circles is a particular case of the following *Proposition*, which we leave it to the reader to prove.
Proposition 2.

If \( \mathcal{C}(O_1, R_1) \) and \( \mathcal{C}(O_2, R_2) \) are two circles such as the power of center \( O_1 \) towards \( \mathcal{C}(O_2, R_2) \) is \(-R_1^2\), then the radical axis of the circles is the perpendicular in \( O_1 \) on the line of centers \( O_1O_2 \).

Bibliography.
