Numerical Result of Supersymmetric Klein-Gordon Equation. Plausible Observation of Supersymmetric-Meson

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In the context of some recent papers suggesting CT-symmetric QM in order to generalize PT-symmetric QM, in this paper we present an idea that there is quite compelling reasoning to argue in favour of supersymmetric extension of Klein-Gordon equation. Its numerical solutions in some simplest conditions are presented. Since the potential corresponding to this supersymmetric KGE is neither Coulomb, Yukawa, nor Hulthen potential [2a], then one can expect to observe a new type of matter, which may be called 'supersymmetric-meson'. Its presence may be expected in particular in the process of breaking of Coulomb barrier in low energy schemes. Further observation is of course recommended in order to refute or verify this proposition.

Introduction

In recent years, there is growing interest on various paths of generalization of supersymmetric extension of Quantum Mechanics, for instance using PT-symmetry [2][6] and CT-symmetry [1]. Interestingly, it can be shown that this CT-symmetry or PT-symmetry yield real eigenvalues, and may also correspond to the zeroes of Riemann zeta function [1]. Therefore, it seems interesting to see whether implications of this new symmetry to some known equations in Quantum Mechanics could yield new observables.

In this context, one can argue that it is possible too to extend Klein-Gordon equation using the hypothesis of PT-symmetry. While this idea has been discussed generally in [2], to our present knowledge its solution has not been presented yet up to this time.

Therefore in the present paper, numerical solutions of this PT-symmetric Klein-Gordon equation in some simplest conditions are presented; in particular we consider solution of Klein-Gordon equation with complex valued time-differential operator. Apart from PT-symmetric considerations, our motivation to consider complex valued Klein-Gordon operator comes from the fact that modified Klein-Gordon correspond to quadratic Dirac equation [5]. Since the potential corresponding to this PT-symmetric KGE is neither Coulomb, Yukawa, nor Hulthen potential [2a], then one can expect to observe a new type of matter, which may be called 'supersymmetric-meson'.

First we will find out numerical solution of (known) standard Klein-Gordon equation, and thereafter we consider its PT-symmetric extension. All numerical computation was performed using Mathematica. [8]

Further observation is of course recommended in order to verify or refute the propositions outlined herein.

Numerical solution of Klein-Gordon equation

First we write down the standard Klein-Gordon equation [3, p.9]:

$$\left(\frac{\hbar^2}{c^2}\frac{\partial^2}{\partial t^2}-\hbar^2\nabla^2+m^2c^2\right)\varphi(x,t)=0\,. \quad (1)$$

Alternatively, one used to assign standard value c=1 and also $\hbar=1$, therefore equation (1) may be written as:

$$\left(\frac{\partial^2}{\partial t^2}-\nabla^2+m^2\right)\varphi(x,t)=0\,, \quad (2)$$

Where the first two terms are often written in the form of square Nabla operator. One simplest version of this equation [3]:

$$-\left(\frac{\partial S_0}{\partial t}\right)^2+m^2=0 \quad (3)$$

yields the known solution:

$$S_0=\pm mt+cons\tan t \quad (4)$$

The equation (3) yields wave equation which describes a particle at rest with positive energy (lower sign) or with negative energy (upper sign). Radial solution of equation (3) yields Yukawa potential which predicts meson as observables.

It is interesting to note here, however, that numerical solution of equation (1), (2) and (3) yield slightly different result, as follows.

- For equation (1) we get.

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-h^2D[#,x,x]+(m^2)(c^2)+(h^2/c^2)D[#,t,t])&[y[x,t]]==0
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$$c^2 m^2+\frac{h^2 y^{(0,2)}[x,t]}{c^2}-h^2 y^{(2,0)}[x,t]==0$$

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DSolve[%,y[x,t],{x,t}]
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$$\left\{\left\{y[x,t]\to\frac{c^2m^2x^2}{2h^2}+C[1]\left[t-\frac{\sqrt{-c^2h^4}\,x}{c^2h^2}\right]+C[2]\left[t+\frac{\sqrt{-c^2h^4}\,x}{c^2h^2}\right]\right\}\right\}$$

- For equation (2) we get.

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( -D[#,x,x]+m^2+D[#,t,t])&[y[x,t]]==0
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$$m^2+y^{(0,2)}[x,t]-y^{(2,0)}[x,t]==0$$

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DSolve[%,y[x,t],{x,t}]
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$$\left\{\left\{y[x,t]\to\frac{m^2x^2}{2}+C[1][t-x]+C[2][t+x]\right\}\right\}$$

- For equation (3) we get.

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( m^2-D[#,t,t])&[y[x,t]]==0
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$$m^2 - y^{(0,2)}[x, t] == 0$$

DSolve[%,y[x,t],{x,t}]

$$\left\{\left\{y[x, t] \to \frac{m^2 t^2}{2} + C[1][x] + t\, C[2][x]\right\}\right\}$$

One may note that this numerical solution is in quadratic form ($\frac{m^2 t^2}{2} + cons\tan t$), therefore it is rather different from equation (4).

Numerical solution of Klein-Gordon equation with complex valued time-differential operator.

As it has been discussed in the context of quaternion Quantum Mechanics [5], it may be useful to consider complex valued Klein-Gordon operator in lieu of standard Nabla operator in equation (2). Therefore, here we rewrite a plausible extension of equation (2) and (3) as follows:

$$\left[\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) + i\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\right]\varphi(x,t) = -m^2\,, \tag{5}$$

And for equation [3] we can write:

$$\left[\left(\frac{\partial S_0}{\partial t}\right)^2 + i\left(\frac{\partial S_0}{\partial t}\right)^2\right] = m^2\,. \tag{6}$$

Numerical solutions for these equations were obtained in similar way with the previous equations:

- For equation (5) we get.

(-D[#,x,x]+D[#,t,t]-I*D[#,x,x]+I*D[#,t,t]+m^2)&[y[x,t]]==0

$$m^2 + (1+i)\, y^{(0,2)}[x, t] - (1+i)\, y^{(2,0)}[x, t] == 0$$

DSolve[%,y[x,t],{x,t}

$$\left\{\left\{y[x, t] \to \left(\frac{1}{4} - \frac{i}{4}\right) m^2 x^2 + C[1][t-x] + C[2][t+x]\right\}\right\}$$

- For equation (6) we get.

(- m^2+D[#,t,t]+I*D[#,t,t])&[y[x,t]]==0

$$-m^2 + (1+i)\, y^{(0,2)}[x, t] == 0$$

DSolve[%,y[x,t],{x,t}]

$$\left\{\left\{y[x, t] \to \left(\frac{1}{4} - \frac{i}{4}\right) m^2 t^2 + C[1][x] + t\,C[2][x]\right\}\right\}$$

At this point one may note that supersymmetric extension of Klein-Gordon equation, in particular by introducing complex-valued differential operator yields quite different solutions compared to known standard solution of Klein-Gordon equation (4), i.e. in the form:

$$y(x,t) = \left(\frac{1}{4} - \frac{i}{4}\right) m^2 t^2 + cons\tan t \tag{7}$$

Since the potential corresponding to this PT-symmetric KGE is neither Coulomb, Yukawa, nor Hulthen potential [2a], then one can expect to observe a new type of matter, which may be called 'supersymmetric-meson'. If this new type of particles can be observed in near future, then it can be regarded as early support to the new hypothesis of PT-symmetric and CT-symmetric as considered by some preceding papers [1][2][6].

In our opinion, its presence may be expected in particular in the process of breaking of Coulomb barrier in low energy schemes.

Further observation is also recommended in order to verify and explore further this proposition.

Concluding remarks

In this paper we present an idea that there is quite compelling reasoning to argue in favour of supersymmetric extension of Klein-Gordon equation. Its numerical solutions in some simplest conditions are presented.

Since the potential corresponding to this supersymmetric KGE is neither Coulomb, Yukawa, or Hulthen potential, then one can expect to observe a new type of matter, which may be called 'supersymmetric-meson'. Its presence may be expected in particular in the process of breaking of Coulomb barrier in low energy schemes. Further observation is of course recommended in order to refute or verify this proposition.

It is recommended to conduct further observation in order to verify and also to explore various implications of our propositions as described herein.

Acknowledgment

The writers would like to thank to Profs. C. Castro & D. Rapoport for valuable discussions.

References

[1] Castro, C., "The Riemann hypothesis is a consequence of CT-invariant Quantum Mechanics," submitted to JMPA, 12 Feb. (2007)

[2] Znojil., M., arXiv:hep-th/0408081 (2004) p.7-8; [2a] math-ph/0002017 (2000)

[3] Kiefer, C., arXiv:gr-qc/9312015 (1993)

[4] Smarandache, F., & V. Christianto, *Progress in Physics* Vol.3 No. 1 (Apr. 2007), http://www.ptep-online.com

[5] Christianto, V., *EJTP* Vol. 3 No. 12 (Oct. 2006) 117-144, http://www.ejtp.com

[6] Dorey, P., *et al*., arXiv:0410013 (2004); [6a] Mostafazadeh, A., arXiv:quant-ph/0304080 (2003)

[7] Smarandache, F., & V. Christianto, *Progress in Physics* Vol.2 No. 4 (Oct. 2006), http://www.ptep-online.com

[8] Toussaint, M., arXiv:cs.SC/0105033 (2001).

First version: 18th Feb. 2007.