Florentin Smarandache, *Neutrosophic Perspectives: Triplets, Duplets, Multisets, Hybrid Operators, Modal Logic, Hedge Algebras. And Applications (Second extended and improved edition), Pons Publishing House, Brussels, 2017; section I.2, pp. 20-24.*

I.2. The Average Positive Qualitative Neutrosophic Function and The Average Negative Qualitative Neutrosophic Function

The *Average Positive Quality Neutrosophic Function* (also known as *Neutrosophic Score Function*, which means expected/average value) of a neutrosophic number.

Let *(t, i, f)* be a single-valued neutrosophic number, where $t, i, f \in [0, 1]$.

The component t (truth) is considered as a positive quality, while i (indeterminacy) and f (falsehood) are considered negative qualities.

Contrarily, $1-t$ is considered a negative quality, while $1-i$ and $1-f$ are considered positive qualities.

Then, the average positive quality function of a neutrosophic number is defined as: (1)

$$s^{+} : [0,1]^3 \rightarrow [0,1], s^{+}(t,i,f) = \frac{t+(1-i)+(1-f)}{3} = \frac{2+t-i-f}{3}.$$

We now introduce for the first time the *Average Negative Quality Neutrosophic Function* of a neutrosophic number, defined as: (2)

$$s^{-}:[0,1]^{3} \to [0,1], s^{-}(t,i,f) = \frac{(1-t)+i+f}{3} = \frac{1-t+i+f}{3}.$$

Theorem I.2.1.

The average positive quality neutrosophic function and the average negative quality neutrosophic function are complementary to each other, or

$$s^{+}(t,i,f) + s^{-}(t,i,f) = 1. \quad (3)$$

Proof.

$$s^{+}(t,i,f) + s^{-}(t,i,f) = \frac{2+t-i-f}{3} + \frac{1-t+i+f}{3} = 1.$$

(4)

The *Neutrosophic Accuracy Function* has been defined by:

$h: [0, 1]^3 \to [-1, 1], h(t, i, f) = t - f.$ (5)

We introduce now for the first time the *Extended Accuracy Neutrosophic Function*, defined as follows:

$h_e: [0, 1]^3 \to [-2, 1], h_e(t, i, f) = t - i - f,$ (6)

which varies on a range: from the worst negative quality (-2) [or minimum value], to the best positive quality [or maximum value].

The *Neutrosophic Certainty Function* is:

$$c: [0, 1]^3 \rightarrow [0, 1], \; c(t, i, f) = t. \qquad (7)$$

Generalization.

The above functions can be extended for the case when the neutrosophic components *t, i, f* are intervals (or, even more general, subsets) of the unit interval *[0, 1]*.

Total Order.

Using three functions from above: neutrosophic score function, neutrosophic accuracy function, and neutrosophic certainty function, one can define a total order on the set of neutrosophic numbers.

In the following way:

Let (t_1, i_1, f_1) and (t_2, i_2, f_2), where $t_1, i_1, f_1, t_2, i_2, f_2 \in [0, 1]$, be two single-valued neutrosophic numbers. Then:

– 1. If $s^+(t_1, i_1, f_1) > s^+(t_2, i_2, f_2)$, then $(t_1, i_1, f_1) >_N (t_2, i_2, f_2)$;

– 2. If $s^+(t_1, i_1, f_1) = s^+(t_2, i_2, f_2)$ and $h(t_1, i_1, f_1) > h(t_2, i_2, f_2)$, then $(t_1, i_1, f_1) >_N (t_2, i_2, f_2)$;

– 3. If $s^+(t_1, i_1, f_1) = s^+(t_2, i_2, f_2)$ and $h(t_1, i_1, f_1) = h(t_2, i_2, f_2)$ and $c(t_1, i_1, f_1) > c(t_2, i_2, f_2)$, then $(t_1, i_1, f_1) >_N (t_2, i_2, f_2)$;

– 4. If $s^+(t_1, i_1, f_1) = s^+(t_2, i_2, f_2)$ and $h(t_1, i_1, f_1) = h(t_2, i_2, f_2)$ and $c(t_1, i_1, f_1) = c(t_2, i_2, f_2)$, then $(t_1, i_1, f_1) = (t_2, i_2, f_2)$.

Applications.

All the above functions are used in the ranking (comparison) of two neutrosophic numbers in multi-criteria decision making.

Example of Comparison of Single-Valued Neutrosophic Numbers.

Let's consider two single-valued neutrosophic numbers: <0.6, 0.1, 0.4> and <0.5, 0.1, 0.3>.

The neutrosophic score functions is:

$s^+(0.6, 0.1, 0.4) = (2 + 0.6 - 0.1 - 0.4) / 3 =$
$= 2.1 / 3 = 0.7;$

$s^+(0.5, 0.1, 0.3) = (2 + 0.5 - 0.1 - 0.3) / 3 =$
$= 2.1 / 3 = 0.7;$

Since $s^+(0.6, 0.1, 0.4) = s^+(0.5, 0.1, 0.3)$, we need to compute the neutrosophic accuracy functions:

$a(0.6, 0.1, 0.4) = 0.6 – 0.4 = 0.2;$

$a(0.5, 0.1, 0.3) = 0.5 – 0.3 = 0.2.$

Since $a(0.6, 0.1, 0.4) = a(0.5, 0.1, 0.3)$, we need to compute the neutrosophic certainty functions:

$c(0.6, 0.1, 0.4) = 0.6;$

c(0.5, 0.1, 0.3) = 0.5.

Because c(0.6, 0.1, 0.4) > c(0.5, 0.1, 0.3), we eventually conclude that the first neutrosophic number is greater than the second neutrosophic number, or:

(0.6, 0.1, 0.4) $>_N$ (0.5, 0.1, 0.3).

So, we need three functions in order to make a total order on the set of neutrosophic numbers.

References

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