Smarandache's Cevians Theorem (I)

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Abstract.
We present the Smarandache's Cevians Theorem in the geometry of the triangle.

Smarandache's Cevians Theorem (I).
In a triangle $\Delta\, ABC$ let's consider the Cevians AA', BB' and CC' that intersect in P. Then:

$$E(P)=\frac{\|PA\|}{\|PA'\|}+\frac{\|PB\|}{\|PB'\|}+\frac{\|PC\|}{\|PC'\|}\geq 6$$

and

$$F(P)=\frac{\|PA\|}{\|PA'\|}\cdot\frac{\|PB\|}{\|PB'\|}\cdot\frac{\|PC\|}{\|PC'\|}\geq 8$$

where $A'\in[BC],\ B'\in[CA],\ C'\in[AB]$.

Proof:
We'll apply the theorem of Van Aubel three times for the triangle $\Delta\, ABC$, and it results:

$$\frac{\|PA\|}{\|PA'\|}=\frac{\|AC'\|}{\|C'B\|}+\frac{\|AB'\|}{\|B'C\|}$$
$$\frac{\|PB\|}{\|PB'\|}=\frac{\|BA'\|}{\|A'C\|}+\frac{\|BC'\|}{\|C'A\|}$$
$$\frac{\|PC\|}{\|PC'\|}=\frac{\|CA'\|}{\|A'B\|}+\frac{\|CB'\|}{\|B'A\|}$$

If we add these three relations and we use the notation

$$\frac{\|AC'\|}{\|C'B\|}=x>0,\ \frac{\|AB'\|}{\|B'C\|}=y>0,\ \frac{\|BA'\|}{\|A'C\|}=z>0$$

then we obtain:

$$E(P)=\left(x+\frac{1}{y}\right)+\left(x+\frac{1}{y}\right)+\left(z+\frac{1}{z}\right)\geq 2+2+2=6$$

The minimum value will be obtained when $x = y = z = 1$, therefore when P will be the gravitation center of the triangle.

When we multiply the three relations we obtain

$$F(P) = \left(x + \frac{1}{y} \right) \cdot \left(x + \frac{1}{y} \right) \cdot \left(z + \frac{1}{z} \right) \geq 8$$

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Open Problems related to Smarandache's Cevians Theorem (I)

1. Instead of a triangle we may consider a polygon $A_1A_2\ldots A_n$ and the lines A_1A_1', A_2A_2', …, A_nA_n' that intersect in a point P.
   Calculate the minimum value of the expressions:

$$E(P) = \frac{\|PA_1\|}{\|PA_1'\|} + \frac{\|PA_2\|}{\|PA_2'\|} + \ldots + \frac{\|PA_n\|}{\|PA_n'\|}$$

$$F(P) = \frac{\|PA_1\|}{\|PA_1'\|} \cdot \frac{\|PA_2\|}{\|PA_2'\|} \cdot \ldots \cdot \frac{\|PA_n\|}{\|PA_n'\|}$$

2. Then let's generalize the problem in the 3D space, and consider the polyhedron $A_1A_2\ldots A_n$ and the lines A_1A_1', A_2A_2', …, A_nA_n' that intersect in a point P. Similarly, calculate the minimum of the expressions E(P) and F(P).

References:

[1] F. Smarandache, Eight Solved and Eight Open Problems in Elementary Geometry, in arXiv.org.

[2 F. Smarandache, *Problèmes avec et sans… problèmes!*, pp. 49 & 54-60, Somipress, Fés, Morocoo, 1983.