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Smooth Neutrosophic Preuniform Spaces

Abstract

As a new branch of philosophy, the neutrosophy was presented by Smarandache in 1998. It was presented as the study of origin, nature, and scope of neutralities; as well as their interactions with different ideational spectra. The aim of this paper is to introduce the concepts of smooth neutrosophic preuniform space, smooth neutrosophic preuniform subspace, and smooth neutrosophic preuniform mappings. Furthermore, some properties of these concepts will be investigated.

Keywords

Fuzzy sets, neutrosophic sets, smooth neutrosophic preuniformity, smooth neutrosophic preuniform subspace, smooth neutrosophic preuniform mappings.

1. Introduction

In 1984, R. Badard [[3], [4]] introduced the concept of a fuzzy preuniformity and he discussed the links between fuzzy preuniformity and fuzzy pretopology. In 1986, R. Badard [6] introduced the basic idea of smooth structure, Badard et al. [5] (1993) investigated some properties of smooth preuniform. Ramadan et al. [10] (2003) introduced smooth topologies induced by a smooth uniformity and investigated some properties of them. In 1983 the intuitionistic fuzzy set was introduced by Atanassov [[1], [2], [7]], as a generalization of fuzzy sets in Zadeh's sense [16], where besides the degree of membership of each element there was considered a degree of non-membership. Smarandache [[13], [14], [15]], defined the notion of neutrosophic set, which is a generalization of Zadeh's fuzzy sets and Atanassov's intuitionistic fuzzy set. Neutrosophic sets have been investigated by Salama et al. [[11], [12]]. The purpose of this paper is to introduce the concepts of smooth neutrosophic preunifrm space, smooth neutrosophic preuniform subspace, and smooth neutrosophic preuniform mappings. We also investigate some of their properties.

2. Preliminaries

In this section we use X to denote a nonempty set, I to denote the closed unit interval [0, 1], I_0 to denote the interval $(0,1]$, I_1 to denote the interval $[0,1)$, and I^X to be the set of all fuzzy subsets defined on X. By $\underline{0}$ and $\underline{1}$ we denote the characteristic functions of ϕ and X, respectively. The family of all neutrosophic sets in X will be denoted by $\aleph(X)$.

2.1. Definition [14], [15]. A neutrosophic set A (NS for short) on a nonempty set X is defined as: $A = \langle x, T_A(x), I_A(x), F_A(x) \rangle$, $x \in X$ where $T, I, F: X \to [0, 1]$, and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ representing the degree of membership (namely $T_A(x)$), the degree of indeterminacy (namely, $I_A(x)$), and the degree of non-membership (namely $F_A(x)$); for each element $x \in X$ to the set A.

2.2. Definition [13], [14]. The Null (empty) neutrosophic set 0_N and the absolute (universe) neutrosophic set 1_N are defined as follows:

$$\text{Type I}: \ 0_N = \langle x, 0, 0, 1 \rangle, \ x \in X \quad , \quad 1_N = \langle x, 1, 1, 0 \rangle, \ x \in X$$

$$\text{Type II}: \ 0_N = \langle x, 0, 1, 1 \rangle, \ x \in X \quad , \quad 1_N = \langle x, 1, 0, 0 \rangle, \ x \in X$$

2.3. Definition [11], [12]. A neutrosophic set A is a subset of a neutrosophic set B, ($A \subseteq B$), may be defined as:

$$\text{Type I}: \ A \subseteq B \Leftrightarrow T_A(x) \le T_B(x), I_A(x) \le I_B(x), F_A(x) \ge F_B(x), \quad \forall x \in X$$

$$\text{Type II}: \ A \subseteq B \Leftrightarrow T_A(x) \le T_B(x), I_A(x) \ge I_B(x), F_A(x) \ge F_B(x), \quad \forall x \in X$$

2.4. Definition [11], [12]. The Complement of a neutrosophic set A, denoted by coA, is defined as:

$$\text{Type I}: \ coA = \langle x, F_A(x), 1 - I_A(x), T_A(x) \rangle$$

$$\text{Type II}: \ coA = \langle x, 1 - T_A(x), 1 - I_A(x), 1 - F_A(x) \rangle$$

2.5. Definition [11], [12]. Let $A, B \in \aleph(X)$ then:

$$\text{Type I}: \ A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x)) \rangle$$

$$\text{Type II}: \ A \cup B = \langle x, \max(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \min(F_A(x), F_B(x)) \rangle$$

$$\text{Type I}: \ A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x)) \rangle$$

$$\text{Type II}: \ A \cap B = \langle x, \min(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \max(F_A(x), F_B(x)) \rangle$$

$$[\ \ ]A = \langle x, T_A(x), I_A(x), 1 - T_A(x) \rangle \quad , \quad \langle\ \ \rangle A = \langle x, 1 - F_A(x), I_A(x), F_A(x) \rangle$$

2.6. Definition [11], [12]. Let $\{A_i\}, i \in J$ be an arbitrary family of neutrosophic sets, then:

$$\text{Type I}: \ \bigcup_{i \in J} A_i = \left\langle x, \sup_{i \in j} T_{A_i}(x), \sup_{i \in j} I_{A_i}(x), \inf_{i \in j} F_{A_i}(x) \right\rangle$$

$$\text{Type II}: \ \bigcup_{i \in J} A_i = \left\langle x, \sup_{i \in j} T_{A_i}(x), \inf_{i \in j} I_{A_i}(x), \inf_{i \in j} F_{A_i}(x) \right\rangle$$

$$\text{TypeI:}\ \bigcap_{i\in J} A_i = \left\langle x, \inf_{i\in j} T_{A_i}(x), \inf_{i\in j} I_{A_i}(x), \sup_{i\in j} F_{A_i}(x) \right\rangle$$

$$\text{TypeII:}\ \bigcap_{i\in J} A_i = \left\langle x, \inf_{i\in j} T_{A_i}(x), \sup_{i\in j} I_{A_i}(x), \sup_{i\in j} F_{A_i}(x) \right\rangle$$

2.7. Definition [11], [12]. The difference between two neutrosophic sets A and B defined as $A \setminus B = A \cap coB$.

2.8. Definition [11], [12]. Every intuitionistic fuzzy set A on X is NS having the form $A = \langle x, T_A(x), 1-(T_A(x)+F_A(x)), F_A(x) \rangle$, and every fuzzy set A on X is NS having the form $A = \langle x, T_A(x), 0, 1-T_A(x) \rangle$, $x \in X$.

2.9. Definition [8]. Let Y be a subset of X and $A \in I^X$; the restriction of A on Y is denoted by $A_{/Y}$. For each $B \in I^Y$, the extension of B on X, denoted by B_X, is defined by:

$$B_X = \begin{cases} B(x) & \text{if } x \in Y \\ 0.5 & \text{if } x \in X - Y \end{cases}$$

2.10. Definition [6]. A smooth topological space (STS) is an ordered pair (X, τ), where X is a nonempty set and $\tau : I^X \to I$ is a mapping satisfying the following properties:

(O1) $\tau(\underline{0}) = \tau(\underline{1}) = 1$

(O2) $\forall A_1, A_2 \in I^X,\ \tau(A_1 \cap A_2) \ge \tau(A_1) \wedge \tau(A_2)$

(O3) $\forall A_i, i \in J,\ \tau(\bigcup_{i\in J} A_i) \ge \bigwedge_{i\in J} \tau(A_i)$

2.11. Definition [5]. A fuzzy preuniform structure U^* on X is a family of fuzzy sets in $X \times X$, called entourages which satisfies:

(FP_1) $\Delta \subseteq u$, for every $u \in U^*$, where Δ is the diagonal :

$$\Delta(x,y) = \begin{cases} 0 & ,\text{if } x \ne y \\ 1 & ,\text{if } x = y \end{cases} \quad ,\forall x, y \in X$$

(FP_2) If $u \in U^*$ and $u \subseteq v$, then $v \in U^*$ $\quad , \forall\ u, v \in I^{X\times X}$

The pair (X, U^*) is said to be a fuzzy preuniform space.

2.12. Definition [5]. Let (X, U^*) be a fuzzy preuniform space, the following potential properties are considered:

(FP_3) For every $u \in U^*$, we can assert that $u^{-1} \in U^*$, where $u^{-1}(x,y) = u(y,x)$.

In this case, (X, U^*) is said to be symmetrical.

(FP_4) For every $u, v \in U^*$, we can assert that $u \cap v \in U^*$.

In this case, (X, U^*) is said to be of type D.

(FP_5) For every $u \in U^*$, there exists $v \in U^*$ such that $v \Theta v \subseteq u$, where
$v\Theta v(x,y) = \sup\{v(x,z) \wedge v(z,y) : z \in X\}$

In this case, (X, U^*) is said to be of type S.

Note that a fuzzy preunifom space which is symmetrical, of type D and of type S is a fuzzy uniform space as defined by Hutton [9].

2.13. Definition [5]. A smooth preuniform structure U on X is a fuzzy set in the fuzzy sets in $X \times X$. U is an element of $I^{I^{X \times X}}$ which satisfies:

(SP_1) $\Delta \not\subseteq u \Rightarrow U(u) = 0$ for every $u \in I^{X \times X}$

(SP_2) $u \subseteq v \Rightarrow U(v) \geq U(u)$, $\forall$ $u, v \in I^{X \times X}$

(SP_3) $U(X \times X) = 1$, where $(X \times X)(x,y) = 1$, for every $x, y \in X$

The pair (X, U) is said to be a smooth preuniform space.

2.14. Definition [5]. Let (X, U) be a smooth preuniform space, the following potential properties are considered:

(SP_4) For every $u \in I^{X \times X}$, we have $U(u) = U(u^{-1})$, where $u^{-1}(x,y) = u(y,x)$.

In this case, (X, U) is said to be symmetrical.

(SP_5) For every $u, v \in I^{X \times X}$, we have $U(u \cap v) \geq U(u) \wedge U(v)$,
but from (SP_2) we can write $U(u \cap v) = U(u) \wedge U(v)$

In this case, (X, U) is said to be of type D.

(SP_6) If we have $\sup_{v \in I^{X \times X}} \{U(v) : v\Theta v \subseteq u\}) \geq U(u)$, for every $u \in I^{X \times X}$

In this case, (X, U) is said to be of type S.

3. Smooth Neutrosophic Preuniform Spaces

Now, we will define two types of smooth neutrosophic preuniform spaces,

a smooth neutrosophic preuniform space (SNPS) take the form (X, U^T, U^I, U^F) and the mappings $U^T, U^I, U^F : I^{X \times X} \to I$ represent the degree of membership, the degree of indeterminacy, and the degree of non-membership respectively.

3.1. Smooth Neutrosophic preuniform Spaces of type I

3.1.1. Definition. A smooth neutrosophic preuniformity (U^T, U^I, U^F) of type I satisfying the following axioms:

$(SNPI_1)$ $\Delta \not\subseteq u \Rightarrow U^T(u) = U^I(u) = 0$ and $U^F(u) = 1$, for every $u \in I^{X\times X}$

$(SNPI_2)$ $u \subseteq v \Rightarrow U^T(v) \geq U^T(u)$, $U^I(v) \geq U^I(u)$, and $U^F(v) \leq U^F(u)$, $\forall$ $u, v \in I^{X\times X}$ (X, U^T, U^I, U^F)

$(SNPI_3)$ $U^T(X \times X) = U^I(X \times X) = 1$, and $U^F(X \times X) = 0$

is said to be a smooth neutrosophic preuniform space of type I

$(SNPI_4)$ For every $u \in I^{X\times X}$, we have:

$$U^T(u) = U^T(u^{-1}), U^I(u) = U^I(u^{-1}), \text{and } U^F(u) = U^F(u^{-1}).$$

In this case, (X, U^T, U^I, U^F) is said to be symmetrical.

$(SNPI_5)$ For every $u, v \in I^{X\times X}$, we have:

$$U^T(u \cap v) \geq U^T(u) \wedge U^T(v), U^I(u \cap v) \geq U^I(u) \wedge U^I(v), \text{and}$$
$$U^F(u \cap v) \leq U^F(u) \vee U^F(v), \text{but from } (SNPI_2) \text{ we can write}$$
$$U^T(u \cap v) = U^T(u) \wedge U^T(v), U^I(u \cap v) = U^I(u) \wedge U^I(v), \text{and}$$
$$U^F(u \cap v) = U^F(u) \vee U^F(v).$$

In this case, (X, U^T, U^I, U^F) is said to be of type D.

$(SNPI_6)$ If we have:

$$\sup_{v \in I^{X\times X}} \{U^T(v) : v\Theta v \subseteq u\}) \geq U^T(u), \quad \sup_{v \in I^{X\times X}} \{U^I(v) : v\Theta v \subseteq u\}) \geq U^I(u), \text{and}$$
$$\inf_{v \in I^{X\times X}} \{U^F(v) : v\Theta v \subseteq u\}) \leq U^F(u), \qquad \text{for every } u \in I^{X\times X}.$$

In this case, (X, U^T, U^I, U^F) is said to be of type S.

3.1.2. Example. Let $X = \{a, b\}$. Define the mappings $U^T, U^I, U^F : I^{X\times X} \to I$ as:

$$U^T(u) = \begin{cases} 1 & , \text{if } u = X \times X \\ 0.5 & , \text{if } \Delta \subseteq u \\ 0 & , \text{otherwise} \end{cases}$$

$$U^I(u) = \begin{cases} 1 & , \text{if } u = X \times X \\ 0.6 & , \text{if } \Delta \subseteq u \\ 0 & , \text{otherwise} \end{cases}$$

$$U^F(u) = \begin{cases} 0 & , \text{if } u = X \times X \\ 0.3 & , \text{if } \Delta \subseteq u \\ 1 & , \text{otherwise} \end{cases}$$

Then (X, U^T, U^I, U^F) is a smooth neutrosophic preuniform space of type I on X.

3.1.3.Remark. Both U^T and U^I with their conditions are smooth preuniformities.

3.1.4.Proposition. Let $\{(U_i^T, U_i^I, U_i^F)\}_{i \in J}$ be a family of smooth neutrosophic preuniformitiess on X. Then $\bigcup_{i\in J} (U_i^T, U_i^I, U_i^F)$ and $\bigcap_{i\in J} (U_i^T, U_i^I, U_i^F)$ are smooth neutrosophic preuniformities on X.

Proof. First, for $\bigcup_{i\in J}(U_i^T, U_i^I, U_i^F)$:

($SNPI_1$) Let $u \in I^{X\times X}$ and $\Delta \not\subseteq u$. It follows that $U_i^T(u) = U_i^I(u) = 0$ and

$U_i^F(u) = 1$, for every $i \in J$, hence $\sup_{i\in J} U_i^T(u) = \sup_{i\in J} U_i^I(u) = 0$ and $\inf_{i\in J} U_i^F(u) = 1$.

($SNPI_2$) Let $u, v \in I^{X\times X}$ such that $u \subseteq v$. Then for every $i \in J$ we have $U_i^T(v) \geq U_i^T(u)$,

$U_i^I(v) \geq U_i^I(u)$, and $U_i^F(v) \leq U_i^F(u)$, hence $\sup_{i\in J} U_i^T(v) \geq \sup_{i\in J} U_i^T(u)$,

$\sup_{i\in J} U_i^I(v) \geq \sup_{i\in J} U_i^I(u)$, and $\inf_{i\in J} U_i^F(v) \leq \inf_{i\in J} U_i^F(u)$.

($SNPI_3$) $U_i^T(X\times X) = U_i^I(X\times X) = 1$, and $U_i^F(X\times X) = 0$, for every $i \in J$. Then

$\sup_{i\in J} U_i^T(X\times X) = \sup_{i\in J} U_i^I(X\times X) = 1$, and $\inf_{i\in J} U_i^F(X\times X) = 0$.

Second, the proof for $\bigcap_{i\in J}(U_i^T, U_i^I, U_i^F)$ is similar to the first

3.1.5.Proposition. Let $\{(U_i^T, U_i^I, U_i^F)\}_{i\in J}$ be a family of smooth neutrosophic preuniformitiess on X .Then:

(i) If every (U_i^T, U_i^I, U_i^F) is symmetrical, then $\bigcup_{i\in J}(U_i^T, U_i^I, U_i^F)$ and $\bigcap_{i\in J}(U_i^T, U_i^I, U_i^F)$ are also symmetrical.

(ii) If every (U_i^T, U_i^I, U_i^F) is of type D , then $\bigcap_{i\in J}(U_i^T, U_i^I, U_i^F)$ is also of type D

(iii) If every (U_i^T, U_i^I, U_i^F) is of type S , then $\bigcup_{i\in J}(U_i^T, U_i^I, U_i^F)$ is also of type S

Proof. (i) Let $u \in I^{X\times X}$,then $U_i^T(u) = U_i^T(u^{-1})$, $U_i^I(u) = U_i^I(u^{-1})$, and

$U_i^F(u) = U_i^F(u^{-1})$ $\forall$ $i \in J$, hence $\sup_{i\in J} U_i^T(u) = \sup_{i\in J} U_i^T(u^{-1})$, $\sup_{i\in J} U_i^I(u) = \sup_{i\in J} U_i^I(u^{-1})$,

and $\inf_{i\in J} U_i^F(u) = \inf_{i\in J} U_i^F(u^{-1})$, also $\inf_{i\in J} U_i^T(u) = \inf_{i\in J} U_i^T(u^{-1})$, $\inf_{i\in J} U_i^I(u) = \inf_{i\in J} U_i^I(u^{-1})$,

and $\sup_{i\in J} U_i^F(u) = \sup_{i\in J} U_i^F(u^{-1})$

(ii) Let $u, v \in I^{X\times X}$, then $U_i^T(u\cap v) = U_i^T(u)\wedge U_i^T(v), U_i^I(u\cap v) = U_i^I(u)\wedge U_i^I(v)$, and $U_i^F(u\cap v) = U_i^F(u)\vee U_i^F(v)\ \forall\ i\in J$, hence $\inf_{i\in J} U_i^T(u\cap v) = \inf_{i\in J} U_i^T(u)\wedge \inf_{i\in J} U_i^T(v)$, $\inf_{i\in J} U_i^I(u\cap v) = \inf_{i\in J} U_i^I(u)\wedge \inf_{i\in J} U_i^I(v)$, and $\sup_{i\in J} U_i^F(u\cap v) = \sup_{i\in J} U_i^F(u)\vee \sup_{i\in J} U_i^F(v)$.

(iii) Let $\sup_{v\in I^{X\times X}}\{U_i^T(v) : v\Theta v\subseteq u\}\geq U_i^T(u)$, $\sup_{v\in I^{X\times X}}\{U_i^I(v) : v\Theta v\subseteq u\}\geq U_i^I(u)$, and $\inf_{v\in I^{X\times X}}\{U_i^F(v) : v\Theta v\subseteq u\}\leq U_i^F(u), \forall u\in I^{X\times X}$, then $\sup_{v\in I^{X\times X}}\{\sup_{i\in J} U_i^T(v) : v\Theta v\subseteq u\}\geq \sup_{i\in J} U_i^T(u)$, $\sup_{v\in I^{X\times X}}\{\sup_{i\in J} U_i^I(v) : v\Theta v\subseteq u\}\geq \sup_{i\in J} U_i^I(u)$, and $\inf_{v\in I^{X\times X}}\{\inf_{i\in J} U_i^F(v) : v\Theta v\subseteq u\}\leq \inf_{i\in J} U_i^F(u)$.

Next, we will introduce a kind of subspace of a smooth neutrosophic preuniform space and some hereditary properties.

3.1.6.Definition. Let A be a nonempty subset of X and let $u\in I^{A\times A}$. We define the extension of u to $X\times X$, denoted $u_{X\times X}$ by:

$$u_{X\times X}(x,y) = \begin{cases} u(x,y) & ,\text{if } x,y\in A \\ 0.5 & ,\text{ otherwise} \end{cases}$$

3.1.7.Definition. Let A be a nonempty subset of X. We define the subdiagonal $\Delta_A\in I^{X\times X}$ by:

$$\Delta_A(x,y) = \begin{cases} 1 & ,\text{if } x=y\in A \\ 0 & ,\text{ otherwise} \end{cases}$$

One may notice that $(\Delta_A\cup\Delta_{coA}) = \Delta$

3.1.8. Proposition. Let (X,U^T,U^I,U^F) be a smooth neutrosophic preuniform space and let A be a nonempty subset of X, and the mappings $U_A^T, U_A^I, U_A^F : I^{A\times A}\to I$ defined by:

$$U_A^T(u) = \begin{cases} 1 & ,\text{ if } u=A\times A \\ U^T(u_{X\times X}\cup\Delta_{coA}) & ,\ \forall u\in I^{A\times A}\setminus\{A\times A\} \end{cases},$$

$$U_A^I(u) = \begin{cases} 1 & ,\text{ if } u=A\times A \\ U^I(u_{X\times X}\cup\Delta_{coA}) & ,\ \forall u\in I^{A\times A}\setminus\{A\times A\} \end{cases},\text{ and}$$

$$U_A^F(u) = \begin{cases} 0 & ,\text{ if } u=A\times A \\ U^F(u_{X\times X}\cup\Delta_{coA}) & ,\ \forall u\in I^{A\times A}\setminus\{A\times A\} \end{cases},\text{ where}$$

$(A\times A)(x,y)=1$ for every $x,y\in A$. Then (U_A^T,U_A^I,U_A^F) is a smooth neutrosophic preuniformity on A.

Proof. ($SNPI_1$) Let $\Delta^A\in I^{A\times A}$ be the diagonal in A.Then, $\forall\ u\in I^{A\times A}$ we have :

$$\Delta^A\not\subseteq u \Rightarrow \Delta\not\subseteq(u_{X\times X}\cup\Delta_{coA})\Rightarrow U^T(u_{X\times X}\cup\Delta_{coA}) = U^I(u_{X\times X}\cup\Delta_{coA}) = 0$$
$$\text{and } U^F(u_{X\times X}\cup\Delta_{coA}) = 1\Rightarrow U_A^T(u) = U_A^I(u) = 0 \text{ and } U_A^F(u) = 1.$$

(SNPI_2) Let $u, v \in I^{A\times A}$ such that $u \subseteq v$. Then it follows that $(u_{X\times X} \cup \Delta_{coA}) \subseteq$ $(v_{X\times X} \cup \Delta_{coA})$, hence $U^T(v_{X\times X} \cup \Delta_{coA}) \geq U^T(u_{X\times X} \cup \Delta_{coA})$, $U^I(v_{X\times X} \cup \Delta_{coA}) \geq$ $U^I(u_{X\times X} \cup \Delta_{coA})$, and $U^F(v_{X\times X} \cup \Delta_{coA}) \leq U^F(u_{X\times X} \cup \Delta_{coA})$. So, $U_A^T(v) \geq U_A^T(u)$, $U_A^I(v) \geq U_A^I(u)$, and $U_A^F(v) \leq U_A^F(u)$. (SNPI_3) The proof is straightforward from the definition.

3.1.9. Definition. The smooth neutrosophic preuniform space (A, U_A^T, U_A^I, U_A^F) is called a subspace of (X, U^T, U^I, U^F) and (U_A^T, U_A^I, U_A^F) is called the smooth neutrosophic preuniformity on A induced by (U^T, U^I, U^F).

3.1.10. Proposition. Let (U^T, U^I, U^F) be a smooth neutrosophic preuniformity on X, A be a nonempty subset of X and (U_A^T, U_A^I, U_A^F) be the corresponding smooth neutrosophic preuniformity on A induced by (U^T, U^I, U^F). Then the properties (SNPI_4) and (SNTI_5) are hereditary.

Proof. (SNPI_4) Let $u \in I^{A\times A}$. Then it follows:

(1) If $u = A\times A$, we find that $U_A^T(u) = U_A^T(u^{-1}) = U_A^I(u) = U_A^I(u^{-1}) = 1$, and $U_A^F(u) = U_A^F(u^{-1}) = 0$.

(2) If $u \neq A\times A$, we find that $U_A^T(u) = U^T(u_{X\times X} \cup \Delta_{coA}) = U^T((u_{X\times X} \cup \Delta_{coA})^{-1})$

$$= U^T((u_{X\times X})^{-1} \cup (\Delta_{coA})^{-1}) = U^T((u^{-1})_{X\times X} \cup \Delta_{coA}) = U_A^T(u^{-1}), \text{ because}$$

$$(u_{X\times X})^{-1}(x,y) = u_{X\times X}(y,x) = \begin{cases} u(y,x) & ,\text{if } x, y \in A \\ 0.5 & ,\text{ otherwise} \end{cases} = \begin{cases} u^{-1}(x,y) & ,\text{if } x, y \in A \\ 0.5 & ,\text{ otherwise} \end{cases}$$

$$= (u^{-1})_{X\times X}(x,y).$$

Similarly, we can prove that $U_A^I(u) = U_A^I(u^{-1})$ and $U_A^F(u) = U_A^F(u^{-1})$.

(SNPI_5) Let $u, v \in I^{A\times A}$. Then we obtain successively:

$$U_A^T(u \cap v) = U^T((u\cap v)_{X\times X} \cup \Delta_{coA}) = U^T((u_{X\times X} \cup \Delta_{coA}) \cap (v_{X\times X} \cup \Delta_{coA}))$$

$$\geq U^T(u_{X\times X} \cup \Delta_{coA}) \wedge U^T(v_{X\times X} \cup \Delta_{coA})) = U_A^T(u) \wedge U_A^T(v).$$ Similarly, we can prove that $U_A^I(u\cap v) \geq U_A^I(u) \wedge U_A^I(v)$ and $U_A^F(u\cap v) \leq U_A^F(u) \vee U_A^F(v)$.

3.1.11. Definition. Consider two ordinary sets X, Y and a mapping f from X into Y. The fuzzy product $f \otimes f$ is defined as the following mapping:

$$f \otimes f : I^{X\times X} \to I^{Y\times Y}$$

$$u \mapsto (f\otimes f)(u),\ \forall\, u \in I^{X\times X}$$

where $(f\otimes f)(u)$ is defined as the following fuzzy set in $Y\times Y$:

$$(f\otimes f)(u) : Y\times Y \to I$$

$$(y_1, y_2) \mapsto \begin{cases} \sup\{u(x_1,x_2) : x_1, x_2 \in X, f(x_1) = y_1 \text{ and } f(x_2) = y_2\} & ,\text{if } y_1, y_2 \in \mathrm{rng}(f) \\ 0 & ,\text{otherwise} \end{cases}$$

3.1.12. Definition. Consider two ordinary sets X, Y .Let f be a mapping from X to Y and $v \in I^{Y\times Y}$ .Then the inverse image of v under $(f \otimes f)$ is defined as the following fuzzy set in $X \times X$:

$$(f \otimes f)^{-1}(v) : X \times X \to I$$
$$(x_1, x_2) \mapsto v(f \otimes f)(x_1, x_2) = v\,(f(x_1), f(x_2)),\ \forall\ x_1, x_2 \in X$$

3.1.13. Definition. A map $f : X \to Y$ is called weakly smooth neutrosophic preuniform with respect to the smooth neutrosophic preuniformities (U_1^T, U_1^I, U_1^F) on X and (U_2^T, U_2^I, U_2^F) on Y iff for every $v \in I^{Y\times Y}$ we have:

$$U_2^T(v) > 0 \Rightarrow U_1^T((f \otimes f)^{-1}(v)) > 0, U_2^I(v) > 0 \Rightarrow U_1^I((f \otimes f)^{-1}(v)) > 0,$$
$$\text{and } U_2^F(v) < 1 \Rightarrow U_1^F((f \otimes f)^{-1}(v)) < 1.$$

3.1.14. Definition. A map $f : X \to Y$ is called smooth neutrosophic preuniform with respect to the smooth neutrosophic preuniformities (U_1^T, U_1^I, U_1^F) on X and (U_2^T, U_2^I, U_2^F) on Y iff for every $v \in I^{Y\times Y}$ we have:

$$U_1^T((f \otimes f)^{-1}(v)) \geq U_2^T(v), U_1^I((f \otimes f)^{-1}(v)) \geq U_2^I(v), \text{ and } U_1^F((f \otimes f)^{-1}(v)) \leq U_2^F(v).$$

3.1.15. Definition. A map $f : X \to Y$ is called smooth neutrosophic direct preuniform with respect to the smooth neutrosophic preuniformities (U_1^T, U_1^I, U_1^F) on X and (U_2^T, U_2^I, U_2^F) on Y iff for every $u \in I^{X\times X}$ we have:

$$U_2^T((f \otimes f)(u)) \geq U_1^T(u),\ U_2^I((f \otimes f)(u)) \geq U_1^I(u), \text{ and } U_2^F((f \otimes f)(u)) \leq U_1^F(u).$$

3.1.16. Definition. A map $f : X \to Y$ is called a (weakly) smooth neutrosophic homeomorphism with respect to the smooth neutrosophic preuniformities (U_1^T, U_1^I, U_1^F) on X and (U_2^T, U_2^I, U_2^F) on Y iff f is bijective and f , f^{-1} are (weakly) smooth neutrosophic preuniform.

3.1.17. Proposition. Let (X, U_1^T, U_1^I, U_1^F) and (Y, U_2^T, U_2^I, U_2^F) be two smooth neutrosophic preuniform spaces and $f : X \to Y$ a bijective mapping. The following statements are equivalent:

(i) f is a smooth netrosophic homeomorphism.

(ii)f is smooth netrosophic preuniform and smooth netrosophic direct preuniform

Proof. $(i) \Rightarrow (ii)$ .Let f be a smooth neutrosophic homeomorphism, then f is smooth neutrosophic preuniform, and for every $u \in I^{X\times X}$ we have:

$$U_2^T((f^{-1} \otimes f^{-1})^{-1}(u)) \geq U_1^T(u), U_2^I((f^{-1} \otimes f^{-1})^{-1}(u)) \geq U_1^I(u),$$
$$\text{and } U_2^F((f^{-1} \otimes f^{-1})^{-1}(u)) \leq U_1^F(u).$$

Applying the definitions and from the bijectivity of f we obtain the following result for $y_1, y_2 \in Y$:

$$((f^{-1} \otimes f^{-1})^{-1}(u)(y_1, y_2) = u(f^{-1} \otimes f^{-1})(y_1, y_2) = u(f^{-1}(y_1), f^{-1}(y_2)) = u(x_1, x_2)$$
$$= ((f \otimes f)(u))(y_1, y_2). \text{ So } U_2^T((f \otimes f)(u)) \geq U_1^T(u), U_2^I((f \otimes f)(u)) \geq U_1^I(u), \text{ and}$$
$$U_2^F((f \otimes f)(u)) \leq U_1^F(u), \text{ hence } f \text{ is smooth neutrosoph ic direct preuniform}.$$

(ii) $\Rightarrow$(i) Let f be smooth neutrosophic preuniform and smooth neutrosophic direct preuniform, then for every $u \in I^{X\times X}$ we have:

$$U_2^T((f^{-1}\otimes f^{-1})^{-1}(u)) = U_2^T((f\otimes f)(u)) \geq U_1^T(u), U_2^I((f^{-1}\otimes f^{-1})^{-1}(u)) =$$
$$U_2^I((f\otimes f)(u)) \geq U_1^I(u), \text{ and } U_2^F((f^{-1}\otimes f^{-1})^{-1}(u)) = U_2^F((f\otimes f)(u)) \leq U_1^F(u).$$

Because f is bijective, then f^{-1} is smooth neutrosophic preuniform, hence f is a smooth neutrosophic homeomorphism.

3.1.18. Proposition. Let $f: X \to Y$ be a bijective and smooth neutrosophic direct preuniform mapping with respect to the smooth neutrosophic preuniformities (U^T, U^I, U^F) on X and (U'^T, U'^I, U'^F) on Y and let A be a nonempty subset of X, then the restriction mapping

$$f_{/A}: (A, U_A^T, U_A^I, U_A^F) \to (f(A), U'^T_{f(A)}, U'^I_{f(A)}, U'^F_{f(A)})$$

Is smooth neutrosophic direct preuniform.

Proof. For every $u \in I^{A\times A}$ we have:

$$U'^T_{f(A)}(((f_{/A})\otimes(f_{/A}))(u)) = U'^T((((f_{/A})\otimes(f_{/A}))(u))_{Y\times Y} \cup \Delta_{cof(A)}),$$
$$U'^I_{f(A)}(((f_{/A})\otimes(f_{/A}))(u)) = U'^I((((f_{/A})\otimes(f_{/A}))(u))_{Y\times Y} \cup \Delta_{cof(A)}),$$
$$U'^F_{f(A)}(((f_{/A})\otimes(f_{/A}))(u)) = U'^F((((f_{/A})\otimes(f_{/A}))(u))_{Y\times Y} \cup \Delta_{cof(A)}),$$
$$U_A^T(u) = U^T(u_{X\times X} \cup \Delta_{coA}) \leq U'^T((f\otimes f)(u_{X\times X} \cup \Delta_{coA})),$$
$$U_A^I(u) = U^I(u_{X\times X} \cup \Delta_{coA}) \leq U'^I((f\otimes f)(u_{X\times X} \cup \Delta_{coA})), \text{ and}$$
$$U_A^F(u) = U^F(u_{X\times X} \cup \Delta_{coA}) \geq U'^F((f\otimes f)(u_{X\times X} \cup \Delta_{coA})).$$

Applying the definitions and from the bijectivity of f we obtain the following result for $y_1, y_2 \in Y$:

$$((((f_{/A})\otimes(f_{/A}))(u))_{Y\times Y} \cup \Delta_{cof(A)})(y_1, y_2)$$
$$= \begin{cases} 1 & ,\text{if } y_1 = y_2 \in cof(A) \\ ((f_{/A})\otimes(f_{/A}))(u)(y_1, y_2) & ,\text{if } y_1, y_2 \in f(A) \\ 0.5 & ,\text{otherwise} \end{cases}$$
$$= \begin{cases} 1 & ,\text{if } f(x_1) = y_1, = f(x_2) = y_2 \text{ and } x_1 = x_2 \in coA \\ u(x_1, x_2) & ,\text{if } f(x_1) = y_1, = f(x_2) = y_2 \text{ and } x_1, x_2 \in A \\ 0.5 & ,\text{otherwise} \end{cases}$$
$$= ((f\otimes f)(u_{X\times X} \cup \Delta_{coA}))(y_1, y_2).$$

So, $U'^T_{f(A)}(((f_{/A})\otimes(f_{/A}))(u)) \geq U_A^T(u)$, $U'^I_{f(A)}(((f_{/A})\otimes(f_{/A}))(u)) \geq U_A^I(u)$, and $U'^F_{f(A)}(((f_{/A})\otimes(f_{/A}))(u)) \leq U_A^F(u)$, hence $f_{/A}$ is smooth neutrosophic direct preuniform.

3.2. Smooth Neutrosophic Preuniform Spaces of type II

In this part we will consider the definitions of type II. In a similar way as in type I, we can state the following definitions and propositions. The proofs of the propositions in type II will be similar to the proofs of the propositions in type I.

3.2.1. Definition. A smooth neutrosophic preuniformity (U^T, U^I, U^F) of type II satisfying the following axioms:

$(SNPII_1)$ $\Delta \not\subseteq u \Rightarrow U^T(u) = 0$ and $U^I(u) = U^F(u) = 1$, for every $u \in I^{X \times X}$

$(SNPII_2)$ $u \subseteq v \Rightarrow U^T(v) \geq U^T(u), U^I(v) \leq U^I(u)$, and $U^F(v) \leq U^F(u), \forall u, v \in I^{X \times X}$ (X, U^T, U^I, U^F)

$(SNPII_3)$ $U^T(X \times X) = 1$, and $U^I(X \times X) = U^F(X \times X) = 0$

is said to be a smooth neutrosophic preuniform space of type II. Also, for type II:

$(SNPII_4)$ For every $u \in I^{X \times X}$, we have:

$$U^T(u) = U^T(u^{-1}), U^I(u) = U^I(u^{-1}), \text{and } U^F(u) = U^F(u^{-1}).$$

In this case, (X, U^T, U^I, U^F) is said to be symmetrical.

$(SNPII_5)$ For every $u, v \in I^{X \times X}$, we have:

$$U^T(u \cap v) \geq U^T(u) \wedge U^T(v), U^I(u \cap v) \leq U^I(u) \vee U^I(v), \text{and}$$
$$U^F(u \cap v) \leq U^F(u) \vee U^F(v), \text{but from } (SNPII_2) \text{ we can write}$$
$$U^T(u \cap v) = U^T(u) \wedge U^T(v), U^I(u \cap v) = U^I(u) \vee U^I(v), \text{and}$$
$$U^F(u \cap v) = U^F(u) \vee U^F(v).$$

In this case, (X, U^T, U^I, U^F) is said to be of type D.

$(SNPII_6)$ If we have:

$$\sup_{v \in I^{X \times X}} \{U^T(v) : v \Theta v \subseteq u\}) \geq U^T(u), \quad \inf_{v \in I^{X \times X}} \{U^I(v) : v \Theta v \subseteq u\}) \leq U^I(u), \text{and}$$
$$\inf_{v \in I^{X \times X}} \{U^F(v) : v \Theta v \subseteq u\}) \leq U^F(u), \qquad \text{for every } u \in I^{X \times X}.$$

In this case, (X, U^T, U^I, U^F) is said to be of type S.

3.2.2. Example. Let $X = \{a, b\}$. Define the mappings $U^T, U^I, U^F : I^{X \times X} \to I$ as:

$$U^T(u) = \begin{cases} 1 & , \text{if } u = X \times X \\ 0.5 & , \text{if } \Delta \subseteq u \\ 0 & , \text{otherwise} \end{cases}$$

$$U^I(u) = \begin{cases} 0 & , \text{if } u = X \times X \\ 0.4 & , \text{if } \Delta \subseteq u \\ 1 & , \text{otherwise} \end{cases}$$

$$U^F(u) = \begin{cases} 0 & , \text{if } u = X \times X \\ 0.7 & , \text{if } \Delta \subseteq u \\ 1 & , \text{otherwise} \end{cases}$$

Then (X, U^T, U^I, U^F) is a smooth neutrosophic preuniform space of type II on X.

3.2.3. Remark. U^T with its conditions is smooth preuniformity.

Note that: the propositions (3.1.4) and (3.1.5) are satisfied for type II.

Proposition.

Let (X, U^T, U^I, U^F) be a smooth neutrosophic preuniform space and let A be a nonempty subset of X, and the mappings $U_A^T, U_A^I, U_A^F : I^{A\times A} \to I$ defined by:

$$U_A^T(u) = \begin{cases} 1 & , \text{if } u = A\times A \\ U^T(u_{X\times X} \cup \Delta_{coA}) & , \forall u \in I^{A\times A} \setminus \{A\times A\} \end{cases},$$

$$U_A^I(u) = \begin{cases} 0 & , \text{if } u = A\times A \\ U^I(u_{X\times X} \cup \Delta_{coA}) & , \forall u \in I^{A\times A} \setminus \{A\times A\} \end{cases},$$

and $$U_A^F(u) = \begin{cases} 0 & , \text{if } u = A\times A \\ U^F(u_{X\times X} \cup \Delta_{coA}) & , \forall u \in I^{A\times A} \setminus \{A\times A\} \end{cases}.$$

Then (U_A^T, U_A^I, U_A^F) is a smooth neutrosophic preuniformity on A.

Proof. Similar to the procedure used to prove proposition (3.1.8).

Also, (A, U_A^T, U_A^I, U_A^F) is a subspace of (X, U^T, U^I, U^F) and (U_A^T, U_A^I, U_A^F) is called the smooth neutrosophic preuniformity on A induced by (U^T, U^I, U^F).

Note that: the proposition (3.1.10) is satisfied for type II.

For smooth neutrosophic preuniform mappings in type II we can state the following definitions:

Definition.

A map $f : X \to Y$ is called weakly smooth neutrosophic preuniform with respect to the smooth neutrosophic preuniformities (U_1^T, U_1^I, U_1^F) on X and (U_2^T, U_2^I, U_2^F) on Y iff for every $v \in I^{Y\times Y}$ we have:

$$U_2^T(v) > 0 \Rightarrow U_1^T((f\otimes f)^{-1}(v)) > 0, U_2^I(v) < 1 \Rightarrow U_1^I((f\otimes f)^{-1}(v)) < 1, \text{ and}$$
$$U_2^F(v) < 1 \Rightarrow U_1^F((f\otimes f)^{-1}(v)) < 1.$$

3.2.6. Definition. A map $f : X \to Y$ is called smooth neutrosophic preuniform with respect to the smooth neutrosophic preuniformities (U_1^T, U_1^I, U_1^F) on X and (U_2^T, U_2^I, U_2^F) on Y iff for every $v \in I^{Y\times Y}$ we have:

$$U_1^T((f\otimes f)^{-1}(v)) \geq U_2^T(v), U_1^I((f\otimes f)^{-1}(v)) \leq U_2^I(v), \text{ and } U_1^F((f\otimes f)^{-1}(v)) \leq U_2^F(v).$$

3.2.7. Definition. A map $f : X \to Y$ is called smooth neutrosophic direct preuniform with respect to the smooth neutrosophic preuniformities (U_1^T, U_1^I, U_1^F) on X and (U_2^T, U_2^I, U_2^F) on Y iff for every $u \in I^{X\times X}$ we have:

$$U_2^T((f\otimes f)(u)) \geq U_1^T(u), U_2^I((f\otimes f)(u)) \leq U_1^I(u), \text{ and } U_2^F((f\otimes f)(u)) \leq U_1^F(u).$$

Note that the definition (3.1.16), and the propositions (3.1.17) , (3.1.18) are satisfied for type II.

4. Conclusion

In this paper, the concepts of smooth neutrosophic preuniform structures were introduced. In two different types we've presented the concepts of smooth neutrosophic preuniform space, smooth neutrosophic preuniform subspace, smooth neutrosophic preuniform mappings. Due to unawareness of the behaviour of the degree of indeterminacy, we've chosen for U^I to act like U^T in the first type, while in the second type we preferred that U^I behaves like U^F.Therefore, the definitions given above can also be modified in several ways depending on the behaviour of U^I.

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