Soft Neutrosophic Group

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Abstract.In this paper we extend the neutrosophic group and subgroup to soft neutrosophic group and soft neutrosophic subgroup respectively. Properties and theorems related to them are proved and many examples are given.

Keywords: Neutrosophic group,neutrosophic subgroup,soft set,soft subset,soft group,soft subgroup,soft neutrosophic group, soft ,neutrosophic subgroup.

1 Introduction

The concept of neutrosophic set was first introduced by Smarandache [13,16] which is a generalization of the classical sets, fuzzy set [18], intuitionistic fuzzy set [4] and interval valued fuzzy set [7]. Soft Set theory was initiated by Molodstov as a new mathematical tool which is free from the problems of parameterization inadequacy. In his paper [11], he presented the fundamental results of new theory and successfully applied it into several directions such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, theory of probability. Later on many researchers followed him and worked on soft set theory as well as applications of soft sets in decision making problems and artificial intelligence. Now, this idea has a wide range of research in many fields, such as databases [5, 6], medical diagnosis problem [7], decision making problem [8], topology [9], algebra and so on.Maji gave the concept of neutrosophic soft set in [8] and later on Broumi and Smarandache defined intuitionistic neutrosophic soft set. We have worked with neutrosophic soft set and its applications in group theory.

2 Preliminaries

2.1 Nuetrosophic Groups

Definition 1 $[14]$ *Let* $(G,*)$ *be any group and let* $\langle G\cup I\rangle=\{a+bI : a,b\in G\}$ *. Then neutrosophic group is generated by* I *and* G *under* $*$ *denoted by* $N(G)=\{\langle G\cup I\rangle,*\}$. I *is called the neutrosophic element with the property* $I^2=I$ *. For an integer* n *,* $n+I$ *and* nI *are neutrosophic elements and* $0.I=0$.

I^{-1} , the inverse of I is not defined and hence does not exist.

Theorem 1 $[14]$ *Let* $N(G)$ *be a neutrosophic group. Then*

1) $N(G)$ in general is not a group;
2) $N(G)$ always contains a group.

Definition 2 *A pseudo neutrosophic group is defined as a neutrosophic group, which does not contain a proper subset which is a group.*

Definition 3 *Let* $N(G)$ *be a neutrosophic group. Then,*

1) A proper subset $N(H)$ of $N(G)$ is said to be a neutrosophic subgroup of $N(G)$ if $N(H)$ is a neutrosophic group, that is, $N(H)$ contains a proper subset which is a group.
2) $N(H)$ is said to be a pseudo neutrosophic subgroup if it does not contain a proper subset which is a group.

Example 1 $(N(Z),+)$, $(N(Q),+)$ $(N(R),+)$ *and* $(N(C),+)$ *are neutrosophic groups of integer, rational, real and complex numbers, respectively.*

Example 2 *Let* $Z_7=\{o,1,2,...,6\}$ *be a group under addition modulo* 7 .

$N(G)=\{\langle Z_7\cup I\rangle,'+'\operatorname{mod}ulo7\}$ *is a neutrosophic group which is in fact a group. For* $N(G)=\{a+bI : a,b\in Z_7\}$ *is a group under `* $+$ *' modulo* 7 .

Definition 4 *Let* $N(G)$ *be a finite neutrosophic group. Let* P *be a proper subset of* $N(G)$ *which under the* *operations of* $N(G)$ *is a neutrosophic group. If* $o(P) / o(N(G))$ *then we call* P *to be a Lagrange neutrosophic subgroup.*

Definition 5 $N(G)$ *is called weakly Lagrange neutrosophic group if* $N(G)$ *has at least one Lagrange neutrosophic subgroup.*

Definition 6 $N(G)$ *is called Lagrange free neutrosophic group if* $N(G)$ *has no Lagrange neutrosophic subgroup.*

Definition7 *Let* $N(G)$ *be a finite neutrosophic group. Suppose* L *is a pseudo neutrosophic subgroup of* $N(G)$ *and if* $o(L) / o(N(G))$ *then we call* L *to be a pseudo Lagrange neutrosophic subgroup.*

Definition 8 *If* $N(G)$ *has at least one pseudo Lagrange neutrosophic subgroup then we call* $N(G)$ *to be a weakly pseudo Lagrange neutrosophic group.*

Definition 9 *If* $N(G)$ *has no pseudo Lagrange neutrosophic subgroup then we call* $N(G)$ *to be pseudo Lagrange free neutrosophic group.*

Definition 10 *Let* $N(G)$ *be a neutrosophic group. We say a neutrosophic subgroup* H *of* $N(G)$ *is normal if we can find* x *and* y *in* $N(G)$ *such that* $H = xHy$ *for all* $x, y \in N(G)$ *(Note* $x = y$ *or* $y = x^{-1}$ *can also occur).*

Definition 11 *A neutrosophic group* $N(G)$ *which has no nontrivial neutrosophic normal subgroup is called a simple neutrosophic group.*

Definition 12 *Let* $N(G)$ *be a neutrosophic group. A proper pseudo neutrosophic subgroup* P *of* $N(G)$ *is said to be normal if we have* $P = xPy$ *for all* $x, y \in N(G)$ *. A neutrosophic group is said to be pseudo simple neutrosophic group if* $N(G)$ *has no nontrivial pseudo normal subgroups.*

2.2 Soft Sets

Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U , and $A \subset E$. Molodtsov [12] defined the soft set in the following manner:

Definition13 $[11]$ *A pair* (F, A) *is called a soft set over* U *where* F *is a mapping given by* F : $A \to P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U . For $e \in A$, $F(e)$ may be considered as the set of e -elements of the soft set (F, A) , or as the set of e-approximate elements of the soft set.

Example 3 *Suppose that* U *is the set of shops.* E *is the set of parameters and each parameter is a word or sentence. Let*

$$E = \left\{ \begin{array}{l} \text{high rent,normal rent,} \\ \text{in good condition,in bad condition} \end{array} \right\}.$$

Let us consider a soft set (F, A) *which describes the attractiveness of shops that Mr.* Z *is taking on rent. Suppose that there are five houses in the universe* $U = \{h_1, h_2, h_3, h_4, h_5\}$ *under consideration, and that* $A = \{e_1, e_2, e_3\}$ *be the set of parameters where*

e_1 stands for the parameter 'high rent,

e_2 stands for the parameter 'normal rent,

e_3 stands for the parameter 'in good condition.

Suppose that

$F(e_1) = \{h_1, h_4\}$,

$F(e_2) = \{h_2, h_5\}$,

$F(e_3) = \{h_3, h_4, h_5\}$.

The soft set (F, A) is an approximated family $\{F(e_i), i = 1, 2, 3\}$ of subsets of the set U which gives us a collection of approximate description of an object. Thus, we have the soft set (F, A) as a collection of approximations as below:

$$(F, A) = \{ \text{ high rent } = \{h_1, h_4\}, \text{ normal rent } = \{h_2, h_5\}, \text{ in good condition } = \{h_3, h_4, h_5\}\} .$$

Definition 14 $[3]$. *For two soft sets* (F, A) *and*

(H,B) *over* U , (F,A) *is called a soft subset of* (H,B) *if*

1) $A \subseteq B$ and
2) $F(e) \subseteq H(e)$, for all $e \in A$.

This relationship is denoted by $(F,A) \subset (H,B)$. Similarly (F,A) is called a soft superset of (H,B) if (H,B) is a soft subset of (F,A) which is denoted by $(F,A) \supset (H,B)$.

Definition 15 $[3]$. *Two soft sets* (F,A) *and* (H,B) *over* U *are called soft equal if* (F,A) *is a soft subset of* (H,B) *and* (H,B) *is a soft subset of* (F,A) .

Definition 16 *Let* $[3]$ (F,A) *and* (G,B) *be two soft sets over a common universe* U *such that* $A \cap B \neq \phi$. *Then their restricted intersection is denoted by* $(F,A) \cap_R (G,B) = (H,C)$ *where* (H,C) *is defined as* $H(c) = F(c) \cap G(c)$ *for all* $c \in C = A \cap B$.

Definition 17 $[3]$ *The extended intersection of two soft sets* (F,A) *and* (G,B) *over a common universe* U *is the soft set* (H,C) *, where* $C = A \cup B$ *, and for all* $e \in C$, $H(e)$ *is defined as*

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cap G(e) & \text{if } e \in A \cap B. \end{cases}$$

We write $(F,A) \cap_\varepsilon (G,B) = (H,C)$.

Definition 18 $[3]$ *The restricted union of two soft sets* (F,A) *and* (G,B) *over a common universe* U *is the soft set* (H,C) *, where* $C = A \cup B$ *, and for all* $e \in C$, $H(e)$ *is defined as the soft set* $(H,C) = (F,A) \cup_R (G,B)$ *where* $C = A \cap B$ *and* $H(c) = F(c) \cup G(c)$ *for all* $c \in C$.

Definition 19 $[3]$ *The extended union of two soft sets* (F,A) *and* (G,B) *over a common universe* U *is the soft set* (H,C) *, where* $C = A \cup B$ *, and for all* $e \in C$, $H(e)$ *is defined as*

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

We write $(F,A) \cup_\varepsilon (G,B) = (H,C)$.

2.3 Soft Groups

Definition 20 $[2]$ *Let* (F,A) *be a soft set over* G . *Then* (F,A) *is said to be a soft group over* G *if and only if* $F(x) \prec G$ *forall* $x \in A$.

Example 4 *Suppose that* $G = A = S_3 = \{e,(12),(13),(23),(123),(132)\}$. *Then* (F,A) *is a soft group over* S_3 *where*

$$\begin{aligned} &F(e) = \{e\}, \\ &F(12) = \{e,(12)\}, \\ &F(13) = \{e,(13)\}, \\ &F(23) = \{e,(23)\}, \\ &F(123) = F(132) = \{e,(123),(132)\}. \end{aligned}$$

Definition 21 $[2]$ *Let* (F,A) *be a soft group over* G . *Then*

1) (F,A) is said to be an identity soft group over G if $F(x) = \{e\}$ for all $x \in A$, where e is the identity element of G and
2) (F,A) is said to be an absolute soft group if $F(x) = G$ for all $x \in A$.

Definition 22 *The restricted product* (H,C) *of two soft groups* (F,A) *and* (K,B) *over* G *is denoted by the soft set* $(H,C) = (F,A)\overset{\wedge}{\circ}(K,B)$ *where* $C = A \cap B$ *and* H *is a set valued function from* C

to $P(G)$ *and is defined as* $H(c) = F(c)K(c)$ *for all* $c \in C$ *. The soft set* (H,C) *is called the restricted soft product of* (F,A) *and* (K,B) *over* G.

3 Soft Neutrosophic Group

Definition 23 *Let* $N(G)$ *be a neutrosophic group and* (F,A) *be soft set over* $N(G)$ *.Then* (F,A) *is called soft neutrosophic group over* $N(G)$ *if and only if* $F(x) \prec N(G)$ *, for all* $x \in A$.

Example 5 *Let*

$$N(Z_4) = \left\{ \begin{matrix} 0,1,2,3,I,2I,3I,1+I,1+2I,1+3I, \\ 2+I,2+2I,2+3I,3+I,3+2I,3+3I \end{matrix} \right\}$$

be a neutrosophic group under addition modulo 4. *Let* $A = \{e_1, e_2, e_3, e_4\}$ *be the set of parameters, then* (F,A) *is soft neutrosophic group over* $N(Z_4)$ *where*

$$F(e_1) = \{0,1,2,3\}, F(e_2) = \{0,I,2I,3I\},$$
$$F(e_3) = \{0,2,2I,2+2I\},$$
$$F(e_4) = \{0,I,2I,3I,2,2+2I,2+I,2+3I\}.$$

Theorem 2 *Let* (F,A) *and* (H,A) *be two soft neutrosophic groups over* $N(G)$ *. Then their intersection* $(F,A) \cap (H,A)$ *is again a soft neutrosophic group over* $N(G)$ *.*

Proof The proof is straightforward.

Theorem 3 *Let* (F,A) *and* (H,B) *be two soft neutrosophic groups over N(G). If* $A \cap B = \phi$ *, then* $(F,A) \cup (H,B)$ *is a soft neutrosophic group over* $N(G)$ *.*

Theorem 4 *Let* (F,A) *and* (H,A) *be two soft neutrosophic groups over* $N(G)$ *. If* $F(e) \subseteq H(e)$ *for all* $e \in A$ *, then* (F,A) *is a soft neutrosophic subgroup of* (H,A) *.*

Theorem 5 *The extended union of two soft neutrosophic groups* (F,A) *and* (K,B) *over* $N(G)$ *is not a soft neutrosophic group over* $N(G)$ *.*

Proof Let (F,A) and (K,B) be two soft neutrosophic groups over $N(G)$. Let $C = A \cup B$, then for all $e \in C, (F,A) \cup_{\varepsilon} (K,B) = (H,C)$ where

$$H(e) \begin{cases} = F(e) & \text{If } e \in A - B, \\ = K(e) & \text{If } e \in B - A, \\ = F(e) \cup K(e) & \text{If } e \in A \cap B. \end{cases}$$

As union of two subgroups may not be again a subgroup. Clearly if $e \in C = A \cap B$, then $H(e)$ may not be a subgroup of $N(G)$. Hence the extended union (H,C) is not a soft neutrosophic group over $N(G)$.

Example 6 *Let* (F,A) *and* (K,B) *be two soft neutrosophic groups over* $N(Z_2)$ *under addition modulo* 2 *, where*

$$F(e_1) = \{0,1\}, F(e_2) = \{0,I\}$$

And

$$K(e_2) = \{0,1\}, K(e_3) = \{0,1+I\}.$$

Then clearly their extended union is not a soft neutrosophic group as $H(e_2) = F(e_2) \cup K(e_2) = \{0,1,I\}$ *is not a subgroup of* $N(Z_2)$ *.*

Theorem 6 *The extended intersection of two soft neutrosophic groups over* $N(G)$ *is soft neutrosophic group over* $N(G)$ *.*

Theorem 7 *The restricted union of two soft neutrosophic groups* (F,A) *and* (K,B) *over* $N(G)$ *is not a soft neutrosophic group over* $N(G)$*.*

Theorem 8 *The restricted intersection of two soft neutrosophic groups over* $N(G)$ *is soft neutrosophic group over* $N(G)$ *.*

Theorem 9 *The restricted product of two soft neutrosoph-*

ict groups (F,A) *and* (K,B) *over* $N(G)$ *is a soft neutrosophic group over* $N(G)$.

Theorem 10 *The* AND *operation of two soft neutrosophic groups over* $N(G)$ *is soft neutrosophic group over* $N(G)$.

Theorem 11 *The* OR *operation of two soft neutrosophic groups over* $N(G)$ *may not be a soft neutrosophic group.*

Definition 24 *A soft neutrosophic group which does not contain a proper soft group is called soft pseudo neutrosophic group.*

Example 7 *Let* $N(Z_2) = \langle Z_2 \cup I \rangle = \{0,1,I,1+I\}$ *be a neutrosophic group under addition modulo* $2.$ *Let* $A = \{e_1,e_2,e_3\}$ *be the set of parameters, then* (F,A) *is a soft pseudo neutrosophic group over* $N(G)$ *where*

$$F(e_1) = \{0,1\},$$
$$F(e_2) = \{0,I\},$$
$$F(e_3) = \{0,1+I\}.$$

Theorem 12 *The extended union of two soft pseudo neutrosophic groups* (F,A) *and* (K,B) *over* $N(G)$ *is not a soft pseudo neutrosophic group over* $N(G)$.

Example 8 *Let* $N(Z_2) = \langle Z_2 \cup I \rangle = \{0,1,I,1+I\}$ *be a neutrosophic group under addition modulo* $2.$ *Let* (F,A) *and* (K,B) *be two soft pseudo neutrosophic groups over* $N(G)$*, where*

$$F(e_1) = \{0,1\}, F(e_2) = \{0,I\},$$
$$F(e_3) = \{0,1+I\}.$$

And

$$K(e_1) = \{0,1+I\}, K(e_2) = \{0,1\}.$$

Clearly their restricted union is not a soft pseudo neutrosophic group as union of two subgroups is not a subgroup.

Theorem 13 *The extended intersection of two soft pseudo neutrosophic groups* (F,A) *and* (K,B) *over* $N(G)$ *is again a soft pseudo neutrosophic group over* $N(G)$.

Theorem 14 *The restricted union of two soft pseudo neutrosophic groups* (F,A) *and* (K,B) *over* $N(G)$ *is not a soft pseudo neutrosophic group over* $N(G)$.

Theorem 15 *The restricted intersection of two soft pseudo neutrosophic groups* (F,A) *and* (K,B) *over* $N(G)$ *is again a soft pseudo neutrosophic group over* $N(G)$.

Theorem 16 *The restricted product of two soft pseudo neutrosophic groups* (F,A) *and* (K,B) *over* $N(G)$ *is a soft pseudo neutrosophic group over* $N(G)$.

Theorem 17 *The* AND *operation of two soft pseudo neutrosophic groups over* $N(G)$ *soft pseudo neutrosophic soft group over* $N(G)$.

Theorem 18 *The* OR *operation of two soft pseudo neutrosophic groups over* $N(G)$ *may not be a soft pseudo neutrosophic group.*

Theorem19 *Every soft pseudo neutrosophic group is a soft neutrosophic group.*

Proof The proof is straight forward.

Remark 1 *The converse of above theorem does not hold.*

Example 9 *Let* $N(Z_4)$ *be a neutrosophic group and* (F,A) *be a soft neutrosophic group over* $N(Z_4)$. *Then*

$$F(e_1) = \{0,1,2,3\}, F(e_2) = \{0,I,2I,3I\},$$
$$F(e_3) = \{0,2,2I,2+2I\}.$$

But (F,A) *is not a soft pseudo neutrosophic group as* (H,B) *is clearly a proper soft subgroup of* (F,A). *where*

$$H(e_1) = \{0,2\}, H(e_2) = \{0,2\}.$$

Theorem 20 (F,A) *over* $N(G)$ *is a soft pseudo* *neutrosophic group if* $N(G)$ *is a pseudo neutrosophic group.*

Proof Suppose that $N(G)$ be a pseudo neutrosophic group, then it does not contain a proper group and for all $e \in A,$ the soft neutrosophic group (F,A) over $N(G)$ is such that $F(e) \prec N(G).$ Since each $F(e)$ is a pseudo neutrosophic subgroup which does not contain a proper group which make (F,A) is soft pseudo neutrosophic group.

Example 10 *Let* $N(Z_2) = \langle Z_2 \cup I \rangle = \{0,1,I,1+I\}$ *be a pseudo neutrosophic group under addition modulo* $2.$ *Then clearly* (F,A) *a soft pseudo neutrosophic soft group over* $N(Z_2)$ *, where*

$$F(e_1) = \{0,1\}, F(e_2) = \{0,I\},$$
$$F(e_3) = \{0,1+I\}.$$

Definition 25 *Let* (F,A) *and* (H,B) *be two soft neutrosophic groups over* $N(G)$ *. Then* (H,B) *is a soft neutrosophic subgroup of* (F,A) *, denoted as* $(H,B) \prec (F,A)$ *, if*

1) $B \subset A$ and
2) $H(e) \prec F(e)$, for all $e \in A$.

Example 11 *Let* $N(Z_4) = \langle Z_4 \cup I \rangle$ *be a soft neutrosophic group under addition modulo* 4 *, that is*

$$N(Z_4) = \begin{Bmatrix} 0,1,2,3,I,2I,3I,1+I,1+2I,1+3I, \\ 2+I,2+2I,2+3I,3+I,3+2I,3+3I \end{Bmatrix}.$$

Let (F,A) *be a soft neutrosophic group over* $N(Z_4)$ *, then*

$$F(e_1) = \{0,1,2,3\}, F(e_2) = \{0,I,2I,3I\},$$
$$F(e_3) = \{0,2,2I,2+2I\},$$
$$F(e_4) = \{0,I,2I,3I,2,2+2I,2+I,2+3I\}.$$

(H,B) is a soft neutrosophic subgroup of (F,A), where

$$H(e_1) = \{0,2\}, H(e_2) = \{0,2I\},$$
$$H(e_4) = \{0,I,2I,3I\}.$$

Theorem 21 *A soft group over* G *is always a soft neutrosophic subgroup of a soft neutrosophic group over* $N(G)$ *if* $A \subset B.$

Proof Let (F,A) be a soft neutrosophic group over $N(G)$ and (H,B) be a soft group over $G.$ As $G \subset N(G)$ and for all $b \in B, H(b) \prec G \subset N(G)$. This implies $H(e) \prec F(e)$, for all $e \in A$ as $B \subset A.$ Hence $(H,B) \prec (F,A).$

Example 12 *Let* (F,A) *be a soft neutrosophic group over* $N(Z_4)$ *, then*

$$F(e_1) = \{0,1,2,3\}, F(e_2) = \{0,I,2I,3I\},$$
$$F(e_3) = \{0,2,2I,2+2I\}.$$

Let $B = \{e_1, e_3\}$ *such that* $(H,B) \prec (F,A)$ *, where*

$$H(e_1) = \{0,2\}, H(_3) = \{0,2\}.$$

Clearly $B \subset A$ *and* $H(e) \prec F(e)$ *for all* $e \in B.$

Theorem 22 *A soft neutrosophic group over* $N(G)$ *always contains a soft group over G.*

Proof The proof is followed from above Theorem.

Definition 26 *Let* (F,A) *and* (H,B) *be two soft pseudo neutrosophic groups over* $N(G)$ *. Then* (H,B) *is called soft pseudo neutrosophic subgroup of* (F,A) *, denoted as* $(H,B) \prec (F,A)$ *, if*

1) $B \subset A$
2) $H(e) \prec F(e)$, for all $e \in A$.

Example 13 *Let* (F,A) *be a soft pseudo neutrosophic group over* $N(Z_4)$ *, where*

$$F\left(e_1\right)=\left\{0,I,2I,3I\right\},F\left(e_2\right)=\left\{0,2I\right\}.$$

Hence $\left(H,B\right)\prec\left(F,A\right)$ where

$$H\left(e_1\right)=\left\{0,2I\right\}.$$

Theorem 23 *Every soft neutrosophic group $\left(F,A\right)$ over $N\left(G\right)$ has soft neutrosophic subgroup as well as soft pseudo neutrosophic subgroup.*

Proof Straightforward.

Definition 27 *Let $\left(F,A\right)$ be a soft neutrosophic group over $N\left(G\right)$, then $\left(F,A\right)$ is called the identity soft neutrosophic group over $N\left(G\right)$ if $F\left(x\right)=\left\{e\right\}$, for all $x\in A$, where e is the identity element of G.*

Definition 28 *Let $\left(H,B\right)$ be a soft neutrosophic group over $N\left(G\right)$, then $\left(H,B\right)$ is called Full-soft neutrosophic group over $N\left(G\right)$ if $F\left(x\right)=N\left(G\right)$, for all $x\in A$.*

Example 14 *Let*

$$N(R)=\left\{\begin{matrix} a+bI \ : \ a,b\in R \text{ and} \\ I \text{ is indeterminacy}\end{matrix}\right\}$$

is a neutrosophic real group where R is set of real numbers and $I^2=I$, therefore $I^n=I$, for n a positive integer. Then $\left(F,A\right)$ is a Full-soft neutrosophic real group where

$$F(e)=N(R), \text{ for all } e\in A$$

Theorem 24 *Every Full-soft neutrosophic group contain absolute soft group.*

Theorem 25 *Every absolute soft group over G is a soft neutrosophic subgroup of Full-soft neutrosophic group over $N\left(G\right)$.*

Theorem 26 *Let $N\left(G\right)$ be a neutrosophic group. If order of $N\left(G\right)$ is prime number, then the soft neutrosophic group $\left(F,A\right)$ over $N\left(G\right)$ is either identity soft neutrosophic group or Full-soft neutrosophic group.*

Proof Straightforward.

Definition 29 *Let $\left(F,A\right)$ be a soft neutrosophic group over $N\left(G\right)$. If for all $e\in A$, each $F\left(e\right)$ is Lagrange neutrosophic subgroup of $N\left(G\right)$, then $\left(F,A\right)$ is called soft Lagrange neutrosophic group over $N\left(G\right)$.*

Example 15 *Let $N\left(Z_3/\left\{0\right\}\right)=\{1,2,I,2I\}$ is a neutrosophic group under multiplication modulo 3. Now $\left\{1,2\right\}$, $\left\{1,I\right\}$ are subgroups of $N\left(Z_3/\left\{0\right\}\right)$ which divides order of $N\left(Z_3/\left\{0\right\}\right)$. Then the soft neutrosophic group*

$$\left(F,A\right)=\left\{F\left(e_1\right)=\left\{1,2\right\},F\left(e_2\right)=\left\{1,I\right\}\right\}$$

is an example of soft Lagrange neutrosophic group.

Theorem 27 *If $N\left(G\right)$ is Lagrange neutrosophic group, then $\left(F,A\right)$ over $N\left(G\right)$ is soft Lagrange neutrosophic group but the converse is not true in general.*

Theorem 28 *Every soft Lagrange neutrosophic group is a soft neutrosophic group.*

Proof Straightforward.

Remark 2 *The converse of the above theorem does not hold.*

Example 16 *Let $N(G)=\{1,2,3,4,I,2I,3I,4I\}$ be a neutrosophic group under multiplication modulo 5 and $\left(F,A\right)$ be a soft neutrosophic group over $N\left(G\right)$, where*

$$F\left(e_1\right)=\left\{1,4,I,2I,3I,4I\right\},F\left(e_2\right)=\left\{1,2,3,4\right\},$$
$$F\left(e_3\right)=\left\{1,I,2I,3I,4I\right\}.$$

But clearly it is not soft Lagrange neutrosophic group as $F\left(e_1\right)$ which is a subgroup of $N\left(G\right)$ does not divide order of $N\left(G\right)$.

Theorem 29 *If $N\left(G\right)$ is a neutrosophic group, then the soft Lagrange neutrosophic group is a soft neutrosophic group.*

Proof Suppose that $N\left(G\right)$ be a neutrosophic group and $\left(F,A\right)$ be a soft Lagrange neutrosophic group over $N\left(G\right)$. Then by above theorem $\left(F,A\right)$ is also soft neutrosophic group.

Example 17 *Let* $N(Z_4)$ *be a neutrosophic group and* (F,A) *is a soft Lagrange neutrosophic group over* $N(Z_4)$ *under addition modulo* 4 *, where*

$$F(e_1)=\{0,1,2,3\}, F(e_2)=\{0,I,2I,3I\},$$
$$F(e_3)=\{0,2,2I,2+2I\}.$$

But (F,A) *has a proper soft group* (H,B) *, where* $H(e_1)=\{0,2\}, H(e_3)=\{0,2\}$.

Hence (F,A) is soft neutrosophic group.

Theorem 30 *Let* (F,A) *and* (K,B) *be two soft Lagrange neutrosophic groups over* $N(G)$ *. Then*

1) Their extended union $(F,A)\cup_{\varepsilon}(K,B)$ over $N(G)$ is not soft Lagrange neutrosophic group over $N(G)$.
2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,B)$ over $N(G)$ is not soft Lagrange neutrosophic group over $N(G)$.
3) Their restricted union $(F,A)\cup_{R}(K,B)$ over $N(G)$ is not soft Lagrange neutrosophic group over $N(G)$.
4) Their restricted intersection $(F,A)\cap_{R}(K,B)$ over $N(G)$ is not soft Lagrange neutrosophic group over $N(G)$.
5) Their restricted product $(F,A)\overset{\wedge}{\circ}(K,B)$ over $N(G)$ is not soft Lagrange neutrosophic group over $N(G)$.

Theorem 31 *Let* (F,A) *and* (H,B) *be two soft Lagrange neutrosophic groups over* $N(G)$ *.Then*

1) Their AND operation $(F,A)\wedge(K,B)$ is not soft Lagrange neutrosophic group over $N(G)$.
2) Their OR operation $(F,A)\vee(K,B)$ is not a soft Lagrange neutrosophic group over $N(G)$.

Definition 30 *Let* (F,A) *be a soft neutrosophic group over* $N(G)$ *. Then* (F,A) *is called soft weakly Lagrange neutrosophic group if atleast one* $F(e)$ *is a Lagrange neutrosophic subgroup of* $N(G)$ *, for some* $e\in A$.

Example 18 *Let* $N(G)=\{1,2,3,4,I,2I,3I,4I\}$ *be a neutrosophic group under multiplication modulo* 5 *, then* (F,A) *is a soft weakly Lagrange neutrosophic group over* $N(G)$ *, where*

$$F(e_1)=\{1,4,I,2I,3I,4I\}, F(e_2)=\{1,2,3,4\},$$
$$F(e_3)=\{1,I,2I,3I,4I\}.$$

As $F(e_1)$ *and* $F(e_3)$ *which are subgroups of* $N(G)$ *do not divide order of* $N(G)$.

Theorem 32 *Every soft weakly Lagrange neutrosophic group* (F,A) *is soft neutrosophic group.*

Remark 3 *The converse of the above theorem does not hold in general.*

Example 19 *Let* $N(Z_4)$ *be a neutrosophic group under addition modulo* 4 *and* $A=\{e_1,e_2\}$ *be the set of parameters, then* (F,A) *is a soft neutrosophic group over* $N(Z_4)$ *, where*

$$F(e_1)=\{0,I,2I,3I\}, F(e_2)=\{0,2I\}.$$

But not soft weakly Lagrange neutrosophic group over $N(Z_4)$.

Definition 31 *Let* (F,A) *be a soft neutrosophic group over* $N(G)$ *. Then* (F,A) *is called soft Lagrange free neutrosophic group if* $F(e)$ *is not Lagrangeneutrosophic subgroup of* $N(G)$ *, for all* $e\in A$.

Example 20 *Let* $N(G)=\{1,2,3,4,I,2I,3I,4I\}$ *be a neutrosophic group under multiplication modulo* 5

and then (F, A) *be a soft Lagrange free neutrosophic group over* $N(G)$ *, where*

$$F(e_1) = \{1, 4, I, 2I, 3I, 4I\}, F(e_2) = \{1, I, 2I, 3I, 4I\}.$$

As $F(e_1)$ *and* $F(e_2)$ *which are subgroups of* $N(G)$ *do not divide order of* $N(G)$.

Theorem 33 *Every soft Lagrange free neutrosophic group* (F, A) *over* $N(G)$ *is a soft neutrosophic group but the converse is not true.*

Definition 32 *Let* (F, A) *be a soft neutrosophic group over* $N(G)$ *. If for all* $e \in A$ *, each* $F(e)$ *is a pseudo Lagrange neutrosophic subgroup of* $N(G)$ *, then* (F, A) *is called soft pseudo Lagrange neutrosophic group over* $N(G)$.

Example 21 *Let* $N(Z_4)$ *be a neutrosophic group under addition modulo* 4 *and* $A = \{e_1, e_2\}$ *be the set of parameters, then* (F, A) *is a soft pseudo Lagrange neutrosophic group over* $N(Z_4)$ *where*

$$F(e_1) = \{0, I, 2I, 3I\}, F(e_2) = \{0, 2I\}.$$

Theorem 34 *Every soft pseudo Lagrange neutrosophic group is a soft neutrosophic group but the converse may not be true.*

Proof Straightforward.

Theorem 35 *Let* (F, A) *and* (K, B) *be two soft pseudo Lagrange neutrosophic groups over* $N(G)$ *. Then*

1) Their extended union $(F, A) \cup_{\varepsilon} (K, B)$ over $N(G)$ is not a soft pseudo Lagrange neutrosophic group over $N(G)$.
2) Their extended intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $N(G)$ is not pseudo Lagrange neutrosophic soft group over $N(G)$.
3) Their restricted union $(F, A) \cup_R (K, B)$ over $N(G)$ is not pseudo Lagrange neutrosophic soft group over $N(G)$.
4) Their restricted intersection $(F, A) \cap_R (K, B)$ over $N(G)$ is also not soft pseudo Lagrange neutrosophic group over $N(G)$.
5) Their restricted product $(F, A)^{\wedge}_{\circ} (K, B)$ over $N(G)$ is not soft pseudo Lagrange neutrosophic group over $N(G)$.

Theorem 36 *Let* (F, A) *and* (H, B) *be two soft pseudo Lagrange neutrosophic groups over* $N(G)$ *. Then*

1) Their AND operation $(F, A) \wedge (K, B)$ is not soft pseudo Lagrange neutrosophic group over $N(G)$.
2) Their OR operation $(F, A) \vee (K, B)$ is not a soft pseudo Lagrange neutrosophic soft group over $N(G)$.

Definition 33 *Let* (F, A) *be a soft neutrosophic group over* $N(G)$ *. Then* (F, A) *is called soft weakly pseudo Lagrange neutrosophic group if atleast one* $F(e)$ *is a pseudo Lagrange neutrosophic subgroup of* $N(G)$ *, for some* $e \in A$.

Example 22 *Let* $N(G) = \{1, 2, 3, 4, I, 2I, 3I, 4I\}$ *be a neutrosophic group under multiplication modulo* 5 *Then* (F, A) *is a soft weakly pseudo Lagrange neutrosophic group over* $N(G)$ *, where*

$$F(e_1) = \{1, I, 2I, 3I, 4I\}, F(e_2) = \{1, I\}.$$

As $F(e_1)$ *which is a subgroup of* $N(G)$ *does not divide order of* $N(G)$.

Theorem 37 *Every soft weakly pseudo Lagrange neutrosophic group* (F, A) *is soft neutrosophic group.*

Remark 4 *The converse of the above theorem is not true in general.*

Example 23 *Let* $N(Z_4)$ *be a neutrosophic group under addition modulo* 4 *and* $A=\{e_1,e_2\}$ *be the set of parameters, then* (F,A) *is a soft neutrosophic group over* $N(Z_4)$*,where*

$$F(e_1)=\{),I,2I,3I\},F(e_2)=\{0,2I\}.$$

But it is not soft weakly pseudo Lagrange neutrosophic group.

Theorem 38 *Let* (F,A) *and* (K,B) *be two soft weakly pseudo Lagrange neutrosophic groups over* $N(G)$ *. Then*

1) Their extended union $(F,A)\cup_{\varepsilon}(K,B)$ over $N(G)$ is not soft weakly pseudo Lagrange neutrosophic group over $N(G)$.
2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,B)$ over $N(G)$ is not soft weakly pseudo Lagrange neutrosophic group over $N(G)$.
3) Their restricted union $(F,A)\cup_R(K,B)$ over $N(G)$ is not soft weakly pseudo Lagrange neutrosophic group over $N(G)$.
4) Their restricted intersection $(F,A)\cap_R(K,B)$ over $N(G)$ is not soft weakly pseudo Lagrange neutrosophic group over $N(G)$.
5) Their restricted product $(F,A)\overset{\wedge}{\circ}(K,B)$ over $N(G)$ is not soft weakly pseudo Lagrange neutrosophic group over $N(G)$.

Definition 34 *Let* (F,A) *be a soft neutrosophic group over* $N(G)$ *. Then* (F,A) *is called soft pseudo Lagrange free neutrosophic group if* $F(e)$ *is not pseudo Lagrange neutrosophic subgroup of* $N(G)$ *, for all* $e\in A$ *.*

Example 24 *Let* $N(G)=\{1,2,3,4,I,2I,3I,4I\}$ *be a neutrosophic group under multiplication modulo* 5 *Then* (F,A) *is a soft pseudo Lagrange free neutrosophic group over* $N(G)$ *, where*

$$F(e_1)=\{1,I,2I,3I,4I\},F(e_2)=\{1,I,2I,3I,4I\}.$$

As $F(e_1)$ *and* $F(e_2)$ *which are subgroups of* $N(G)$ *do not divide order of* $N(G)$ *.*

Theorem 39 *Every soft pseudo Lagrange free neutrosophic group* (F,A) *over* $N(G)$ *is a soft neutrosophic group but the converse is not true.*

Theorem 40 *Let* (F,A) *and* (K,B) *be two soft pseudo Lagrange free neutrosophic groups over* $N(G)$ *. Then*

1) Their extended union $(F,A)\cup_{\varepsilon}(K,B)$ over $N(G)$ is not soft pseudo Lagrange free neutrosophic group over $N(G)$.
2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,B)$ over $N(G)$ is not soft pseudo Lagrange free neutrosophic group over $N(G)$.
3) Their restricted union $(F,A)\cup_R(K,B)$ over $N(G)$ is not pseudo Lagrange free neutrosophic soft group over $N(G)$.
4) Their restricted intersection $(F,A)\cap_R(K,B)$ over $N(G)$ is not soft pseudo Lagrange free neutrosophic group over $N(G)$.
5) Their restricted product $(F,A)\overset{\wedge}{\circ}(K,B)$ over $N(G)$ is not soft pseudo Lagrange free neutrosophic group over $N(G)$.

Definition 35 *A soft neutrosophic group* (F,A) *over* $N(G)$ *is called soft normal neutrosophic group over*

$N(G)$ *if* $F(e)$ *is a normal neutrosophic subgroup of* $N(G)$*, for all* $e \in A$.

Example 25 *Let* $N(G)=\{e,a,b,c,I,aI,bI,cI\}$ *be a neutrosophic group under multiplicationwhere* $a^2 = b^2 = c^2 = e, bc = cb = a, ac = ca = b, ab = ba = c$. *Then* (F,A) *is a soft normal neutrosophic group over* $N(G)$ *where*

$$F(e_1)=\{e,a,I,aI\},$$
$$F(e_2)=\{e,b,I,bI\},$$
$$F(e_3)=\{e,c,I,cI\}.$$

Theorem 42 *Every soft normal neutrosophic group* (F,A) *over* $N(G)$ *is a soft neutrosophic group but the converse is not true.*

Theorem 42 *Let* (F,A) *and* (H,B) *be two soft normal neutrosophic groups over* $N(G)$ *. Then*

1) Their extended union $(F,A)\cup_{\varepsilon}(K,B)$ over $N(G)$ is not soft normal neutrosophic group over $N(G)$.
2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,B)$ over $N(G)$ is soft normal neutrosophic group over $N(G)$.
3) Their restricted union $(F,A)\cup_{R}(K,B)$ over $N(G)$ is not soft normal neutrosophic group over $N(G)$.
4) Their restricted intersection $(F,A)\cap_{R}(K,B)$ over $N(G)$ is soft normal neutrosophic group over $N(G)$.
5) Their restricted product $(F,A)\overset{\wedge}{\circ}(K,B)$ over $N(G)$ is not soft normal neutrosophic soft group over $N(G)$.

Theorem 43 *Let* (F,A) *and* (H,B) *be two soft normal neutrosophic groups over* $N(G)$ *. Then*

1) Their AND operation $(F,A)\wedge(K,B)$ is soft normal neutrosophic group over $N(G)$.
2) Their OR operation $(F,A)\vee(K,B)$ is not soft normal neutrosophic group over $N(G)$.

Definition 36 *Let* (F,A) *be a soft neutrosophic group over* $N(G)$ *. Then* (F,A) *is called soft pseudo normal neutrosophic group if* $F(e)$ *is a pseudo normal neutrosophic subgroup of* $N(G)$ *, for all* $e \in A$.

Example 26 *Let* $N(Z_2)=\langle Z_2 \cup I\rangle=\{0,1,I,1+I\}$ *be a neutrosophic group under addition modulo* 2 *and let* $A=\{e_1,e_2\}$ *be the set of parameters, then* (F,A) *is soft pseudo normal neutrosophic group over* $N(G)$*, where*

$$F(e_1)=\{0,I\},F(e_2)=\{0,1+I\}.$$

As $F(e_1)$ *and* $F(e_2)$ *are pseudo normal subgroup of* $N(G)$.

Theorem 44 *Every soft pseudo normal neutrosophic group* (F,A) *over* $N(G)$ *is a soft neutrosophic group but the converse is not true.*

Theorem 45 *Let* (F,A) *and* (K,B) *be two soft pseudo normal neutrosophic groups over* $N(G)$ *. Then*

1) Their extended union $(F,A)\cup_{\varepsilon}(K,B)$ over $N(G)$ is not soft pseudo normal neutrosophic group over $N(G)$.
2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,B)$ over $N(G)$ is soft pseudo normal neutrosophic group over $N(G)$.
3) Their restricted union $(F,A)\cup_{R}(K,B)$ over $N(G)$ is not soft pseudo normal neutrosophic

group over $N(G)$.

4) Their restricted intersection $(F,A)\cap_R (K,B)$ over $N(G)$ is soft pseudo normal neutrosophic group over $N(G)$.

5) Their restricted product $(F,A)\overset{\wedge}{\circ}(K,B)$ over $N(G)$ is not soft pseudo normal neutrosophic group over $N(G)$.

Theorem 46 Let (F,A) *and* (K,B) *be two soft pseudo normal neutrosophic groups over* $N(G)$. *Then*

1) Their AND operation $(F,A)\wedge(K,B)$ is soft pseudo normal neutrosophic group over $N(G)$.

2) Their OR operation $(F,A)\vee(K,B)$ is not soft pseudo normal neutrosophic group over $N(G)$.

Definition 37 *Let* $N(G)$ *be a neutrosophic group. Then* (F,A) *is called soft conjugate neutrosophic group over* $N(G)$ *if and only if* $F(e)$ *is conjugate neutrosophic subgroup of* $N(G)$, *for all* $e\in A$.

Example 27 *Let*

$$N(G)=\begin{Bmatrix}0,1,2,3,4,5,I,2I,3I,4I,5I,\\ 1+I,2+I,3+I,...,5+5I\end{Bmatrix}$$

be a neutrosophic group under addition modulo 6 *and let* $P=\{0,3,3I,3+3I\}$ *and* $K=\{0,2,4,2+2I,4+4I,2I,4I\}$ *are conjugate neutrosophic subgroups of* $N(G)$. *Then* (F,A) *is soft conjugate neutrosophic group over* $N(G)$, *where*

$$F(e_1)=\{0,3,3I,3+3I\},$$
$$F(e_2)=\{0,2,4,2+2I,4+4I,2I,4I\}.$$

Theorem 47 *Let* (F,A) *and* (K,B) *be two soft conjugate neutrosophic groups over* $N(G)$. *Then*

1) Their extended union $(F,A)\cup_{\varepsilon}(K,B)$ over $N(G)$ is not soft conjugate neutrosophic group over $N(G)$.

2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,B)$ over $N(G)$ is again soft conjugate neutrosophic group over $N(G)$.

3) Their restricted union $(F,A)\cup_R (K,B)$ over $N(G)$ is not soft conjugate neutrosophic group over $N(G)$.

4) Their restricted intersection $(F,A)\cap_R (K,B)$ over $N(G)$ is soft conjugate neutrosophic group over $N(G)$.

5) Their restricted product $(F,A)\overset{\wedge}{\circ}(K,B)$ over $N(G)$ is not soft conjugate neutrosophic group over $N(G)$.

Theorem 48 *Let* (F,A) *and* (K,B) *be two soft conjugate neutrosophic groups over* $N(G)$. *Then*

1) Their AND operation $(F,A)\wedge(K,B)$ is again soft conjugate neutrosophic group over $N(G)$.

2) Their OR operation $(F,A)\vee(K,B)$ is not soft conjugate neutrosophic group over $N(G)$.

Conclusion

In this paper we extend the neutrosophic group and subgroup,pseudo neutrosophic group and subgroup to soft neutrosophic group and soft neutrosophic subgroup and respectively soft pseudo neutrosophic group and soft pseudo neutrosophic subgroup. The normal neutrosophic subgroup is extended to soft normal neutrosophic subgroup. We showed all these by giving various examples in order to illustrate the soft part of the neutrosophic notions used.

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