Rajesh Singh

Department of Mathematics, SRM University

Delhi NCR, Sonepat- 131029, India

Sachin Malik

Department of Statistics, Banaras Hindu University

Varanasi-221005, India

Florentin Smarandache

Department of Mathematics, University of New Mexico

Gallup, NM 87301, USA

Some Ratio and Product Estimators Using Known Value of Population Parameters

Published in:
Sachin Malik, Neeraj Kumar, Florentin Smarandache (Editors)
USES OF SAMPLING TECHNIQUES & INVENTORY CONTROL WITH CAPACITY CONSTRAINTS
Pons Editions, Brussels, Belgium, 2016
ISBN 978-1-59973-484-2
pp. 19 - 29

Abstract

In the present article, we proposed a family of estimators for estimating population means using known value of some population parameters. Khoshnevisan et al. [1] proposed a general family of estimators for estimating population means using known value of some population parameter(s) which after some substitutions led to some ratio and product estimators initially proposed by Sisodia and Dwivedi [2], Singh and Tailor [3], Pandey and Dubey [4], Adewara et al. [5], yadav and Kadilar [6]. The present family of estimators provides us significant improvement over previous families in theory. An empirical study is carried out to judge the merit of the proposed estimator.

Keywords: Ratio Estimator, Product Estimator, Population Parameter, Efficiency, Mean Square Error.

1. Introduction

The problem of estimating the population mean in the presence of an auxiliary variable has been widely discussed in finite population sampling literature. Ratio, product and difference methods of estimation are good examples in this context. Ratio method of estimation is quite effective when there is high positive correlation between study and auxiliary variables. On the other hand, if correlation is negative (high), the product method of estimation can be employed efficiently.

In recent years, a number of research papers on ratio-type, exponential ratio-type and regression-type estimators have appeared, based on different types of transformations. Some important contributions in this area are due to Singh and Tailor [3], Shabbir and Gupta [7,8], Kadilar and Cingi [9,10], Khosnevisan et. al.(2007).

Khoshnevisan et al. [1] defined their family of estimators as

$$t = \bar{y}\left[\frac{a\bar{X}+b}{\alpha(a\bar{x}+b)+(1-\alpha)(a\bar{X}+b)}\right]^{g}$$

where $a(\neq 0)$, b are either real numbers or the functions of the known parameters of the auxiliary variable x such as standard deviation (σ_x), Coefficient of Variation (C_x), Skewness ($\beta_1(x)$), Kurtosis ($\beta_2(x)$) and Correlation Coefficient (ρ).

(i). When α=0, a=0=b, g=0, we have the mean per unit estimator, $t_0 = \bar{y}$ with

$$MSE(t_0) = \left(\frac{N-n}{Nn}\right)\bar{Y}^2 C_y^{\,2} \tag{1.1}$$

(ii). When α=1, a=1, b=0, g=1, we have the usual ratio estimator, $t_1 = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)$ with

$$MSE(t_1) = \left(\frac{N-n}{Nn}\right)\bar{Y}^2 (C_y^{\,2} + C_x^{\,2} - 2\rho\rho_x C_y) \tag{1.2}$$

(iii). When α=1, a=1, b=0, g=-1, we have the usual product estimator, $t_2 = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)$ with

$$MSE(t_2) = \left(\frac{N-n}{Nn}\right)\bar{Y}^2 (C_y^{\,2} + C_x^{\,2} + 2\rho\rho_x C_y) \tag{1.3}$$

(iv). When α=1, a=1, b=C_x, g=1, we have Sisodia and Dwivedi [2] ratio estimator, $t_3 = \bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)$ with

$$MSE(t_3) = \left(\frac{N-n}{Nn}\right)\bar{Y}^2 \left(C_y^{\,2} + \left(\frac{\bar{X}}{\bar{X}+C_x}\right)^2 C_x^{\,2} - 2\left(\frac{\bar{X}}{\bar{X}+C_x}\right)\rho\rho_x C_y\right) \tag{1.4}$$

(v). When α=1, a=1, b=C_x, g=-1

we have Pandey and Dubey [4] product estimator, $t_4 = \bar{y}(\frac{\bar{x}+C_x}{\bar{X}+C_x})$ with

$$MSE(t_4) = (\frac{N-n}{Nn})\bar{Y}^2(C_y{}^2 + (\frac{\bar{X}}{\bar{X}+C_x})^2 C_x{}^2 + 2(\frac{\bar{X}}{\bar{X}+C_x})\rho\rho_x C_y) \tag{1.5}$$

(vi). When α=1, a=1, b= ρ , g=1, we have Singh and Taylor [3] ratio estimator, $t_5 = \bar{y}(\frac{\bar{X}+\rho}{\bar{x}+\rho})$ with

$$MSE(t_5) = (\frac{N-n}{Nn})\bar{Y}^2(C_y{}^2 + (\frac{\bar{X}}{\bar{X}+\rho})^2 C_x{}^2 - 2(\frac{\bar{X}}{\bar{X}+\rho})\rho\rho_x C_y) \tag{1.6}$$

(vii). When α=1, a=1, b= ρ , g=-1, we have Singh and Taylor [3] product estimator, $t_6 = \bar{y}(\frac{\bar{x}+\rho}{\bar{X}+\rho})$ with

$$MSE(t_6) = (\frac{N-n}{Nn})\bar{Y}^2(C_y{}^2 + (\frac{\bar{X}}{\bar{X}+\rho})^2 C_x{}^2 + 2(\frac{\bar{X}}{\bar{X}+\rho})\rho\rho_x C_y) \tag{1.7}$$

There are other ratio and product estimators from these families that are not inferred here but this paper will be limited to those ones that made use of Coefficient of Variation (C_x) and Correlation Coefficient (ρ) since the conclusion obtained here can also be inferred on all others that made use of other population parameters such as the standard deviation (σ_x), Skewness ($\beta_1(x)$) and Kurtosis ($\beta_2(x)$) in the same family.

2. On the Modified Ratio and Product Estimators.

Adopting Adewara (2006), Adewara et al. (2012) proposed the following estimators as

$$t^*_1 = \bar{y}^*(\frac{\bar{X}}{\bar{x}^*}), \tag{2.1}$$

$$t^*_2 = \bar{y}^*(\frac{\bar{x}^*}{\bar{X}}), \tag{2.2}$$

$$t^*_3 = \bar{y}^*(\frac{\bar{X}+C_x}{\bar{x}^*+C_x}), \tag{2.3}$$

$$t^*_4 = \bar{y}^*\left(\frac{\bar{x}^* + C_x}{\bar{X} + C_x}\right), \tag{2.4}$$

$$t^*_5 = \bar{y}^*\left(\frac{\bar{X} + \rho}{\bar{x}^* + \rho}\right) \text{ and} \tag{2.5}$$

$$t^*_6 = \bar{y}^*\left(\frac{\bar{x}^* + \rho}{\bar{X} + \rho}\right), \tag{2.6}$$

Where $\bar{x}^*$ and $\bar{y}^*$ are the sample means of the auxiliary variables and variable of interest yet to be drawn with the relationships (i) $\bar{X} = f\bar{x} + (1-f)\bar{x}^*$ and (ii). $\bar{Y} = f\bar{y} + (1-f)\bar{y}^*$. Srivenkataramana and Srinath [12].

The Mean Square Errors of these estimators t^*_i, i = 1,2, …, 6 are as follows:

(i). $$MSE(t^*_1) = \left(\frac{n}{N-n}\right)^2 MSE(t_1) \tag{2.7}$$

(ii). $$MSE(t^*_2) = \left(\frac{n}{N-n}\right)^2 MSE(t_2) \tag{2.8}$$

(iii). $$MSE(t^*_3) = \left(\frac{n}{N-n}\right)^2 MSE(t_3) \tag{2.9}$$

(iv). $$MSE(t^*_4) = \left(\frac{n}{N-n}\right)^2 MSE(t_4) \tag{2.10}$$

(v). $$MSE(t^*_5) = \left(\frac{n}{N-n}\right)^2 MSE(t_5) \tag{2.11}$$

(vi). $$MSE(t^*_6) = \left(\frac{n}{N-n}\right)^2 MSE(t_6) \tag{2.12}$$

Following Adewara et al [5], Yadav and Kadilar [6] proposed some improved ratio and product estimators for estimating the population mean of the study variable as follows

$$\eta^*_1 = k\bar{y}^*\left(\frac{\bar{X}}{\bar{x}^*}\right), \tag{2.13}$$

$$\eta^*_2 = k\,\bar{y}^*\left(\frac{\bar{x}^*}{\bar{X}}\right), \tag{2.14}$$

$$\eta^*_3 = k\,\bar{y}^*\left(\frac{\bar{X}+C_x}{\bar{x}^*+C_x}\right), \tag{2.15}$$

$$\eta^*_4 = k\,\bar{y}^*\left(\frac{\bar{x}^*+C_x}{\bar{X}+C_x}\right), \tag{2.16}$$

$$\eta^*_5 = k\,\bar{y}^*\left(\frac{\bar{X}+\rho}{\bar{x}^*+\rho}\right) \tag{2.17}$$

$$\eta^*_6 = k\,\bar{y}^*\left(\frac{\bar{x}^*+\rho}{\bar{X}+\rho}\right), \tag{2.18}$$

The mean square error of these estimators η^*_i, i=1,2,…,6 are as follows

$$MSE(\eta^*_1) = \bar{Y}^2\left[h^2\left(k_1^2\lambda C_y^2 + \{3k_1^2 - 2k_1\}\lambda C_x^2 - 2\{2k_1^2 - k_1\}\lambda C_{yx}\right) + \{k_1 - 1\}^2\right] \tag{2.19}$$

$$MSE(\eta^*_2) = \bar{Y}^2\left[h^2\left(k_2^2\lambda C_y^2 + k_2^2\lambda C_x^2 + 2\{2k_1^2 - k_1\}\lambda C_{yx}\right) + \{k_2 - 1\}^2\right] \tag{2.20}$$

$$MSE(\eta^*_3) = \bar{Y}^2\left[h^2\left(k_3^2\lambda C_y^2 + \{3k_3^2 - 2k_3\}\nu_1^2\lambda C_x^2 - 2\nu_1\{2k_3^2 - k_3\}\lambda C_{yx}\right) + \{k_3 - 1\}^2\right] \tag{2.21}$$

$$MSE(\eta^*_4) = \bar{Y}^2\left[h^2\left(k_4^2\lambda C_y^2 + k_4^2\nu_1^2\lambda C_x^2 + 2\nu_1\{2k_4^2 - k_4\}\lambda C_{yx}\right) + \{k_4 - 1\}^2\right] \tag{2.22}$$

$$MSE(\eta^*_3) = \bar{Y}^2\left[h^2\left(k_3^2\lambda C_y^2 + \{3k_3^2 - 2k_3\}\nu_2^2\lambda C_x^2 - 2\nu_2\{2k_5^2 - k_5\}\lambda C_{yx}\right) + \{k_5 - 1\}^2\right] \tag{2.23}$$

$$MSE(\eta^*_4) = \bar{Y}^2\left[h^2\left(k_6^2\lambda C_y^2 + k_4^2\nu_2^2\lambda C_x^2 + 2\nu_2\{2k_6^2 - k_6\}\lambda C_{yx}\right) + \{k_6 - 1\}^2\right] \tag{2.24}$$

Where,

$$\lambda = \frac{N-n}{Nn}, h = \frac{n}{N-n}, C_y^2 = \frac{S_y^2}{\bar{Y}^2}, C_x^2 = \frac{S_x^2}{\bar{X}^2}, C_{yx} = \frac{S_{yx}}{\bar{Y}\bar{X}}, \nu_1 = \frac{\bar{X}}{\bar{X}+C_x}, \nu_1 = \frac{\bar{X}}{\bar{X}+C_x} \text{ and } \rho = \frac{S_{yx}}{S_y S_x}$$

And $k_1 = \dfrac{h^2\left[\lambda C_x^2 - \lambda C_{yx}\right]+1}{h^2\left[3C_x^2\lambda - 4C_{yx}\lambda + \lambda C_y^2\right]+1}, k_2 = \dfrac{h^2\lambda C_{yx}+1}{h^2\left[C_x^2\lambda + 4C_{yx}\lambda + \lambda C_y^2\right]+1},$

$$k_3 = \frac{h^2\left[\lambda\nu_1^2 C_x^2 - \nu_1\lambda C_{yx}\right]+1}{h^2\left[3\nu_1^2 C_x^2\lambda - 4\nu_1 C_{yx}\lambda + \lambda C_y^2\right]+1}, k_4 = \frac{h^2\left[\lambda\nu_1\lambda C_{yx}\right]+1}{h^2\left[3\nu_1^2 C_x^2\lambda + 4\nu_1 C_{yx}\lambda + \lambda C_y^2\right]+1}$$

$$k_5 = \frac{h^2\left[\lambda\nu_2^2 C_x^2 - \nu_2\lambda C_{yx}\right]+1}{h^2\left[3\nu_2^2 C_x^2\lambda - 4\nu_2 C_{yx}\lambda + \lambda C_y^2\right]+1}, \text{and } k_6 = \frac{h^2\left[\lambda\nu_2\lambda C_{yx}\right]+1}{h^2\left[3\nu_2^2 C_x^2\lambda + 4\nu_2 C_{yx}\lambda + \lambda C_y^2\right]+1}$$

3. The Proposed family of estimators

Following Malik Singh [14], we define the following class of estimators for population mean $\overline{Y}$ as

$$t_M = \left\{m_1\bar{y}^* + m_2\left[\overline{X} - \bar{x}^*\right]\right\}\left(\frac{\psi\overline{X}+\delta}{\psi\bar{x}^*+\delta}\right)^{\alpha} \exp\left[\frac{\left(\omega\overline{X}+\mu\right)-\left(\omega\bar{x}^*+\mu\right)}{\left(\omega\overline{X}+\mu\right)+\left(\omega\bar{x}^*+\mu\right)}\right]^{\beta} \tag{3.1}$$

Where m_1 and m_2 are suitably chosen constants. ψ, δ, ω, and μ are either real numbers or function of known parameters of the auxiliary variable. The scalar α and β takes values +1 and -1 for ratio and product type estimators respectively.

To obtain the MSE , let us define

$$\bar{y} = \overline{Y}(1+e_0)\ ,\ \bar{x} = \overline{X}(1+e_1)$$

such that $E(e_i) = 0$, i=0,1 and

$$E(e_0^2) = \lambda C_y^2\ ,\quad E(e_1^2) = \lambda C_x^2,\quad E(e_0e_1) = \lambda\rho C_yC_x$$

expressing equation (3.1) in terms of e's and retaining only terms up to second degree of e's, we have

$$t_M = \left[m_1\overline{Y}(1-he_0) + m_2\overline{X}he_1\right]\left\{\frac{\psi\overline{X}+\delta}{\psi\overline{X}(1-he_1)+\delta}\right\}^{\alpha} \exp\left\{\frac{\omega\overline{X}he_1}{2\omega\overline{X}+2\mu-\omega\overline{X}he_1}\right\}^{\beta}$$

$$= \left[m_1\overline{Y}(1-he_0) + m_2\overline{X}he_1\right]\left\{1-R_1he_1\right\}^{-\alpha} \exp\left\{\beta R_2he_1\left(1-\frac{R_2he_1}{2}+\frac{R_2^2h^2e_1^2}{4}\right)\right\}$$

$$= m_1\overline{Y}\begin{bmatrix}1+\alpha he_1+\dfrac{\alpha(\alpha+1)h^2e_1^2}{2}+\dfrac{\beta he_1}{2}+\dfrac{\alpha\beta h^2e_1^2}{2}+\dfrac{\beta^2h^2e_1^2}{8}+\dfrac{\beta h^2e_1^2}{4}-he_0\\ -\alpha h^2e_0e_1-\dfrac{\beta h^2e_0e_1}{2}\end{bmatrix}$$

$$+m_2\overline{X}\left[he_1+\alpha h^2e_1^2+\frac{\beta h^2e_1^2}{2}\right] \tag{3.2}$$

where, $R_1 = \frac{\psi\overline{X}}{\psi\overline{X}+\delta}, R_2 = \frac{\omega\overline{X}}{\omega\overline{X}+\mu}$

Subtracting $\overline{Y}$ from both the sides of (3.2), we have

$$\left(t_M - \overline{Y}\right) = m_1\overline{Y}\left[1 - he_0 + L_1e_1 + L_2e_1^2 - L_3e_0e_1\right] + m_2\overline{X}\left[he_1 + L_4e_1^2\right] - \overline{Y} \quad (3.3)$$

where,

$$L_1 = \alpha R_1 h + \frac{\beta h R_2}{2}$$

$$L_2 = \frac{\alpha(\alpha+1)h^2R_1^2}{2} + \frac{\alpha\beta h^2 R_1 R_2}{2} + \frac{\beta^2 h^2 R_2^2}{8} + \frac{\beta h^2 R_2^2}{4}$$

$$L_3 = \alpha h^2 R_1 + \frac{\beta h^2 R_2}{2}$$

$$L_4 = \alpha h^2 R_1 + \frac{\beta h^2 R_2}{2}$$

Squaring both sides of (3.3) and neglecting terms of e's having power greater than two, we have

$$MSE\left(t_M\right) = \overline{Y}^2\left[1 + m_1^2T_1 + m_2^2T_2 + 2m_1m_2T_3 - 2m_1T_4 - 2m_2T_5\right] \quad (3.4)$$

where,

$$\left.\begin{aligned}
T_1 &= \overline{Y}^2\left[1 + \lambda h^2C_y^2 + L_1^2\lambda C_x^2 - 2hL_1\lambda\rho C_yC_x + 2L_2\lambda C_x^2 - 2L_3\lambda\rho C_yC_x\right]\\
T_2 &= h^2\lambda\overline{X}^2C_x^2\\
T_3 &= \overline{Y}\,\overline{X}\left[L_4\lambda C_x^2 + L_1\lambda hC_x^2 - h^2\lambda\rho C_yC_x\right]\\
T_4 &= \overline{Y}^2\left[1 + L_2\lambda C_x^2 - L_3\lambda\rho C_yC_x\right]\\
T_5 &= \overline{Y}\,\overline{X}L_4\lambda C_x^2
\end{aligned}\right\}$$

minimization of (3.4) with respect to m_1 and m_2 yields optimum values as

$$m_1 = \frac{\left(T_2T_4 - T_3T_5\right)}{T_1T_2 - T_3^2}, \quad m_2 = \frac{\left(T_1T_5 - T_3T_4\right)}{T_1T_2 - T_3^2}$$

4. Empirical Study:

Population I: Kadilar and Cingi [9]

N = 106, n = 20, $\rho = 0.86, C_y = 5.22, C_x = 2.1, \overline{Y} = 2212.59$ and $\overline{X} = 27421.70$

Population II: Maddala [13]

$N = 16$, $n = 4$, $\rho = -0.6823, C_y = 0.2278,\ C_x = 0.0986,\ \overline{Y} = 7.6375$ and $\overline{X} = 75.4313$

4. Results:

Table 4.1: Showing the estimates obtained for both the Khoshnevisan et al. [1] estimators and Adewara et al. [5] estimators

Estimator	Population I ($\rho > 0$)	Population II ($\rho < 0$)
t_0	5411349	0.5676
t_1	2542740	-
t_2	-	0.3387
t_3	2542893	-
t_4	-	0.3388
t_5	2542803	-
t_6	-	0.3376
t^*_1	137519.8	-
t^*_2	-	0.03763
t^*_3	137528	-
t^*_4	-	0.03765
t^*_5	137523.1	-
t^*_6	-	0.03751

Table 4.2: Showing the estimates obtained for Yadav and Kadilar [6] estimators

Estimator	Population I ($\rho > 0$)	Population II ($\rho < 0$)
η^*_1	136145.37	-
η^*_2	-	0.03762
η^*_3	136138.05	-
η^*_4	-	0.03764
η^*_5	136107.94	-
η^*_6	-	0.03750

Table 4.3: MSE of suggested estimators with different values of constants

									MSE	
m_1	m_2	α	β	ψ	δ	ω	μ	**estimator**	**PopI**	**PopII**
1	0	1	0	1	0	-	-	t^*_1	137519.8	-
1	0	-1	0	1	0	-	-	t^*_2	-	0.03763
1	0	1	0	1	C_x	-	-	t^*_3	137528	-
1	0	-1	0	1	C_x	-	-	t^*_4	-	0.03765
1	0	1	0	1	ρ	-	-	t^*_5	137523.1	-
1	0	-1	0	1	ρ	-	-	t^*_6	-	0.03751
m_1	0	1	0	1	0	-	-	η^*_1	136145.37	-
m_1	0	-1	0	1	0	-	-	η^*_2	-	0.03762

m_1	0	1	0	1	C_x	-	-	η^*_3	136138.05	-
m_1	0	-1	0	1	C_x	-	-	η^*_4	-	0.03764
m_1	0	1	0	1	ρ	-	-	η^*_5	136107.94	-
m_1	0	-1	0	1	ρ	-	-	η^*_6	-	0.03750
m_1	m_2	1	1	1	1	1	1	t_M	75502.23	-
m_1	m_2	-1	-1	1	1	1	1	t_M	-	0.03370

Since conventionally, for ratio estimators to hold, $\rho > 0$ and also for product estimators to hold, $\rho < 0$. Therefore two data sets are used in this paper, one to determine the efficiency of the modified ratio estimators and the other to determine that of the product estimators as stated below.

5. Conclusion

In this paper, we have proposed a new family of estimator for estimating unknown population mean of study variable using auxiliary variable. Expressions for the MSE of the estimator are derived up to first order of approximation. The proposed family of estimator is compared with the several existing estimators in literature. From table 4.3, we observe that the new family of estimators performs better than the other estimators considered in this paper for both of the data sets.

References

[1] Khoshnevisan, M., Singh, R., Chauhan, P., Sawan, N. and Smarandache, F.(2007): A general family of estimators for estimating population mean using known value of some population parameter(s). Far East Jour. of theor. statist. 22(2), 181 – 191.

[2] Sisodia, B.V.S. and Dwivedi, V.K.(1981): A modified ratio estimator using coefficient of variation of auxiliary variable. Journal Ind. Soc. Agril. Statist.,33(2), 13 – 18.

[3] Singh, H.P. and Tailor, R. (2003) : Use of known correlation coefficient in estimating the finite population mean. Statist. in Trans. 6(4). 555 – 560.

[4] Pandey, B.N. and Dubey, Vyas (1988): Modified product estimator using coefficient of variation of auxiliary variate, Assam Statistical Rev., 2(2), 64 – 66

[5] A.A. Adewara, R. Singh, M. Kumar(2012) : Efficiency of some modified ratio and product estimators using known value of some population parameters, International Journal of Applied Science and Technology 2 (2) 76–79.

[6] Yadav, S.K. and Kadilar, C. (2013): Improved class of ratio and product estimators. Applied mathematics and computation.

[7] Shabbir, J., Gupta, S. (2005). : Improved ratio estimators in stratified sampling. *Amer. J. Mathemat. Manag. Sci.* 25:293–311

[8] Shabbir, J. and Gupta, S. (2006) : A new estimator of population mean in stratified sampling, Commun. Stat. Theo. Meth. 35: 1201–1209

[9] Kadilar, C. and Cingi, H. (2004) : Ratio Estimators in Simple Random Sampling. Appl. Mathe. and Comput. 151, 893-902

[10] Kadilar, C. and Cingi, H. (2006).: Ratio estimators for the population variance in sample and stratified random sampling. Appl. Mathe. and Comput. 173, 1047 – 1059.

[11] Adewara, A.A. (2006). : Effects of improving both the auxiliary and variable of interest in ratio and product estimators. Proc. Pakistan Acad. Sci.43(4): 275 – 278.

[12] Srivenkataramana, T. and Srinath, K.P. (1976) : Ratio and Product methods of estimation in sample surveys when the two variables are moderately correlated. Vignana Bharathi 2: 54 – 58

[13] Maddala, G.S. (1977): Econometrics. “McGraw Hills Pub. Co.” New York .

[14] Malik and Singh (2015): Estimation of population mean using information on auxiliary attribute in two-phase sampling. Applied Mathematics and computation, 261 (2015) 114-1