New Types of Soft Sets: HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, and TreeSoft Set

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Abstract

This is a review paper, where we recall the definitions together with practical applications of the Soft Set and its extensions to HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, and TreeSoft Set.

Keywords: Soft Set; HyperSoft Set; IndetermSoft Set; IndetermHyperSoft Set; MultiSoft Set; TreeSoft Set.

1. Introduction

The Soft Set was introduced by Molodtsov [1] in 1999. Further on, the HyperSoft Set (2018), IndetermSoft Set (2022), IndetermHyperSoft Set (2022), and TreeSoft Set (2022) were introduced by Smarandache [2-7].

The MultiSoft Set (2010) was introduced by Alkhazaleh et al. [8]. The soft set and its extensions have many applications in our real world. Many hybrid version of the soft set have been proposed and used, combined with fuzzy or fuzzy extension sets, such as: fuzzy soft set, intuitionistic fuzzy soft set, neutrosophic soft set, picture fuzzy soft set, spherical fuzzy soft set, plithogenic soft set, and similarly for fuzzy hypersoft set, intuitionistic fuzzy hypersoft set, neutrosophic hypersoftsoft set, picture fuzzy hypersoft set, spherical fuzzy hypersoft soft set, plithogenic sift set.

Future research may also investigate and apply the newly forms of soft sets, in combinations with fuzzy and fuzzy extension sets, that will result in fuzzy / intuitionistic fuzzy / neutrosophic / picture fuzzy / spherical fuzzy / Pythagorean fuzzy etc. / plithogenic IndetermSoft / IndetermHyperSoft / TreeSoft Set . respectively.

Let us recall their definitions together with real examples.

2. Definition of Soft Set

Let U be a universe of discourse, $P(U)$ the power set of U, and A a set of attributes. Then, the pair (F, U), where $F: A \rightarrow P(U)$ is called a Soft Set over U.

3. Real Example of Soft Set

Let U = {Helen, George, Mary, Richard} and a set M = {Helen, Mary, Richard} included in U.

Let the attribute be: a = size, and its attribute' values respectively:

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Size = A_1 = {small, medium, tall}.

Let the function be: $F: A_1 \rightarrow P(U)$.

Then, for example:

F(tall) = {Helen, Mary},

which means that both Helen and Mary are tall.

4. Definition of IndetermSoft Set

Let U be a universe of discourse, H a non-empty subset of U, and $P(H)$ be the powerset of H. Let a be an attribute, and A be a set of this attribute-values. Then $F: A \rightarrow P(H)$ is called an IndetermSoft Set if at least one of the bellow occurs:

i) the set A has some indeterminacy;

ii) the sets H or $P(H)$ have some indeterminacy;

iii) the function F has some indeterminacy, i.e. there exist at least an attribute-value v that belongs to A, such that $F(v)$ = indeterminate (unclear, incomplete, conflicting, or not unique).

IndetermSoft Set, as an extension of the classical (determinate) Soft Set, deals with indeterminate data, because there are sources unable to provide exact or complete information on the sets A, H, or $P(H)$, nor on the function F. We did not add any indeterminacy, we found the indeterminacy in our real world. Because many sources give approximate/uncertain/incomplete/conflicting information, not exact information as in the Soft Set, as such we still need to deal with such situations.

Herein, 'Indeterm' stands for 'Indeterminate' (uncertain, conflicting, incomplete, not unique outcome).

Similarly, distinctions between determinate and indeterminate operators are taken into consideration. Afterwards, an IndetermSoft Algebra is built, using a determinate soft operator (joinAND), and three indeterminate soft operators (disjoinOR, exclussiveOR, NOT), whose properties are further on studied.

Smarandache has generalized the Soft Set to the HyperSoft set by transforming the function F into a multi-attribute function, and then he introduced the hybrids of Crisp, Fuzzy, Intuitionistic Fuzzy, Neutrosophic, other fuzzy extensions, and Plithogenic HyperSoft Set.

The classical Soft Set is based on a determinate function (whose values are certain, and unique), but in our world there are many sources that, because of lack of information or ignorance, provide indeterminate (uncertain, and not unique – but hesitant or alternative) information. They can be modeled by operators having some degree of indeterminacy due to the imprecision of our world.

5. Real Example of IndetermSoft Set

Assume a town has many houses.

1) Indeterminacy with respect to the function.

1a) You ask a source:

— What houses have the red color in the town?

The source:

— I am not sure, I think the houses h_1 or h_2.

Therefore, F(red) = h_1 or h_2 (indeterminate / uncertain answer).

1b) You ask again:

— But, what houses are yellow?

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The source:

— I do not know, the only thing I know is that the house h_5 is not yellow because I have visited it.

Therefore, F(yellow) = not h_5 (again indeterminate / uncertain answer).

1c) Another question you ask:

— Then what houses are blue?

The source:

— For sure, either h_8 or h_9.

Therefore, F(blue) = either h_8 or h_9 (again indeterminate / uncertain answer).

2) Indeterminacy with respect to the set H of houses.

You ask the source:

— How many houses are in the town?

The source:

— I never counted them, but I estimate their number to be between 100-120 houses.

3) Indeterminacy with respect to the set A of attributes.

You ask the source:

— What are all colors of the houses?

The source:

— I know for sure that there are houses of colors red, yellow, and blue, but I do not know if there are houses of other colors (?)

This is the IndetermSoft Set.

6. Definition of HyperSoft Set

The soft set was extended to the hypersoft set by transforming the function F into a multi-attribute function. Afterwards, the hybrids of HyperSoft Set with the Crisp, Fuzzy, Intuitionistic Fuzzy, Neutrosophic, other fuzzy extensions, and Plithogenic Set were introduced.

Let U be a universe of discourse, $P(U)$ the power set of U. Let $a_1, a_2, \dots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \dots, A_n$, with $A_i \cap A_j = \Phi$, for $i \neq j$, and i, j in $\{1, 2, \dots, n\}$. Then the pair $(F, A_1 \times A_2 \times \dots \times A_n)$, where $F: A_1 \times A_2 \times \dots \times A_n \rightarrow P(U)$, is called a HyperSoft Set over U.

7. Real Example of HyperSoft Set

Let U = {Helen, George, Mary, Richard} and a set M = {Helen, Mary, Richard} included in U.

Let the attributes be: a_1 = size, a_2 = color, a_3 = gender, a_4 = nationality, and their attributes' values respectively:

Size = A_1 = {small, medium, tall},

Color = A_2 = {white, yellow, red, black},

Gender = A_3 = {male, female},

Nationality = A_4 = {American, French, Spanish, Italian, Chinese}.

Let the function be: $F: A_1 \times A_2 \times A_3 \times A_4 \rightarrow P(U)$.

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Then, for example:

F({tall, white, female, Italian}) = {Helen, Mary}, which means that both Helen and Mary are tall, and white, and female, and Italian.

Notice that this is an extension of the previous Real Example of Soft Set.

8. Definition of IndetermHyperSoft Set

Let U be a universe of discourse, H a non-empty subset of U, and $P(H)$ the powerset of H. Let $a_1, a_2, \dots, a_n$, for $n \geq 1$, be n distinct attributes, whose corresponding attribute-values are respectively the sets $A_1, A_2, \dots, A_n$, with $A_i \cap A_j = \Phi$ for $i \neq j$, and i, j in $\{1, 2, \dots, n\}$. Then the pair $(F, A_1 \times A_2 \times \dots \times A_n)$, where $F: A_1 \times A_2 \times \dots \times A_n \to P(H)$, is called an IndetermHyperSoft Set over U if at least one of the bellow occurs:

i) at least one of the sets $A_1, A_2, \dots, A_n$ has some indeterminacy;

ii) the sets H or $P(H)$ have some indeterminacy;

iii) there exist at least one n-plet $(e_1, e_2, \dots, e_n)$ ε $A_1 \times A_2 \times \dots \times A_n$ such that $F(e_1, a_2, \dots, e_n)$ = indeterminate (unclear, uncertain, conflicting, or not unique).

Similarly, IndetermHyperSoft Set ia an extension of the HyperSoft Set, when there is indeterminate data, or indeterminate functions, or indeterminate sets.

9. Real Example of IndetermHyperSoft Set

Assume a town has many houses.

1) Indeterminacy with respect to the function.

1a) You ask a source:

— What houses are of red color and big size in the town?

The source:

— I am not sure, I think the houses h_1 or h_2.

Therefore, F(red, big) = h_1 or h_2 (indeterminate / uncertain answer).

1b) You ask again:

— But, what houses are yellow and small?

The source:

— I do not know, the only thing I know is that the house h_5 is neither yellow nor small because I have visited it.

Therefore, F(yellow, small) = not h_5 (again indeterminate / uncertain answer).

1c) Another question you ask:

— Then what houses are blue and big?

The source:

— For sure, either h_8 or h_9.

Therefore, F(blue, big) = either h_8 or h_9 (again indeterminate / uncertain answer).

2) Indeterminacy with respect to the set H of houses.

You ask the source:

— How many houses are in the town?

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The source:

— I never counted them, but I estimate their number to be between 100-120 houses.

3) Indeterminacy with respect to the product set $A_1 \times A_2 \times \ldots \times A_n$ of attributes.

You ask the source:

— What are all colors and sizes of the houses?

The source:

— I know for sure that there are houses of colors of red, yellow, and blue, but I do not know if there are houses of other colors (?) About the size, I saw many houses that are small, but I do not remember to have seeing big houses.

This is the IndetermHyperSoft Set.

10. Definition of TreeSoft Set

Let U be a universe of discourse, and H a non-empty subset of U, with $P(H)$ the powerset of H.

Let A be a set of attributes (parameters, factors, etc.),

$A = \{A_1, A_2, \ldots, A_n\}$, for integer $n \geq 1$, where $A_1, A_2, \ldots, A_n$ are considered attributes of first level (since they have one-digit indexes).

Each attribute A_i, $1 \leq i \leq n$, is formed by sub-attributes:

$$A_1 = \{A_{1,1}, A_{1,2}, \ldots\}$$

$$A_2 = \{A_{2,1}, A_{2,2}, \ldots\}$$

$$\ldots\ldots\ldots\ldots\ldots$$

$$A_n = \{A_{n,1}, A_{n,2}, \ldots\}$$

where the above $A_{i,j}$ are sub-attributes (or attributes of second level) (since they have two-digit indexes).

Again, each sub-attribute $A_{i,j}$ is formed by sub-sub-attributes (or attributes of third level):

$$A_{i,j,k}$$

And so on, as much refinement as needed into each application, up to sub-sub-…-sub-attributes (or attributes of m-level (or having m digits into the indexes):

$$A_{i1,i2,\ldots,im}$$

Therefore, a graph-tree is formed, that we denote as Tree(A), whose root is A (considered of level zero), then nodes of level 1, level 2, up to level m.

We call *leaves* of the graph-tree, all terminal nodes (nodes that have no descendants).

Then the TreeSoft Set is:

$$F: P(Tree(A)) \rightarrow P(H)$$

Tree(A) is the set of all nodes and leaves (from level 1 to level m) of the graph-tree, and P(Tree(A)) is the powerset of the Tree(A).

All node sets of the *TreeSoft Set of level m* are:

$$\text{Tree}(A) = \{A_{i1} \mid_{i1} = 1, 2, \ldots\}$$

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The first set is formed by the nodes of level 1, second set by the nodes of level 2, third set by the nodes of level 3, and so on, the last set is formed by the nodes of level *m*. If the graph-tree has only two levels ($m = 2$), then the TreeSoft Set is reduced to a MultiSoft Set [8].

11. Practical Example of TreeSoft Set of Level 3

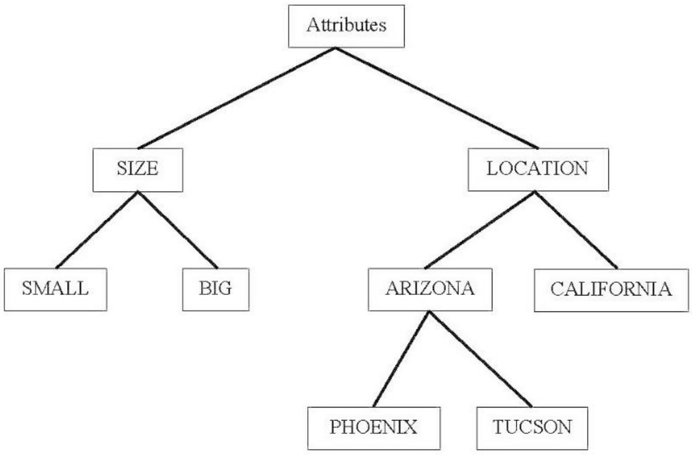

Figure 1: A TreeSoft Set of Level 3

Assume a town has many houses.

This is a classical tree as shown in Figure 1, whose:

Level 0 (the root) is the node Attributes;

Level 1 is formed by the nodes: Size, Location;

Level 2 is formed by the nodes Small, Big, Arizona, California;

Level 3 is formed by the nodes Phoenix, Tucson.

Let's consider $H = \{h_1, h_2, ..., h_{10}\}$ be a set of houses, and $P(H)$ the powerset of H.

And the set of Attributes: $A = \{A_1, A_2\}$, where A_1 = Size, A_2 = Location.

Then $A_1 = \{A_{11}, A_{12}\}$ = {Small, Big}, $A_2 = \{A_{21}, A_{22}\}$ = {Arizona, California} as American states.

Further on, $A_{22} = \{A_{211}, A_{212}\}$ = {Phoenix, Tucson} as Arizonian cities.

Let's assume that the function F gets the following values:

F(Big, Arizona, Phoenix} = { h_9, h_{10} }

F(Big, Arizona, Tucson) = { h_1, h_2, h_3, h_4 }

F(Big, Arizona) = all big houses from both cities, Phoenix and Tucson

= F(Big, Arizona, Phoenix) ∪ F(Big, Arizona, Tucson) = { h_1, h_2, h_3, h_4, h_9, h_{10} }.

12. Conclusion

This paper reviews Soft Set and its extensions to HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, and TreeSoft Set. The HyperSoft Set (2018) is a generalization of Soft Set (1999) and MultiSoft Set (2010), from a uni-variate function to a multi-variate function F; IndetermSoft Set (2022) is an extension of the Soft Set, from the determinate data to indeterminate data; IndetermHyperSoft Set (2022) is an extension of the HyperSoft Set, from the determinate data to indeterminate data; and TreeSoft Set (2022) that is a generalization of the MultiSoft Set.

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