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THE COMBINATION METHOD FOR DEPENDENT EVIDENCE AND ITS APPLICATION FOR SIMULTANEOUS FAULTS DIAGNOSIS

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Abstract:

This paper provides a method based on Dezert-Smarandache Theory (DSmT) for simultaneous faults diagnosis when evidence is dependent. Firstly, according to the characteristics of simultaneous faults, a frame of discernment is given for both single fault and simultaneous faults diagnosis, the DSmT combination rule applicable to simultaneous faults diagnosis is introduced. Secondly, the dependence of original evidence is classified according to three main factors in information acquisition and extraction, a method for evidence decorrelation is provided. On the other hand, the weights for measuring evidence credibility are given to modify independent evidence based on Generalized Ambiguity Measure. Next, DSmT combination rule is used to aggregate the modified evidence. Finally, an example of rotor faults diagnosis is given to illustrate effectiveness of the proposed method.

Keywords:

DSmT; Dependent evidence; Simultaneous faults diagnosis; Membership function; Basic probability assignment

1. Introduction

In fault diagnosis, because of the influence of environment, sensors, information processing technology and other factors, information collected by sensors is usually imprecise, uncertain and incomplete, so the process of fault diagnosis is an uncertainty reasoning and decision-making process in essence. The diversity and complexity of fault modes and features make the traditional method based on single-sensor more and more insufficient, therefore, we can make up the deficiency by using several sensors, which will increase the amount of information and improve the accuracy of output. In fact, fault diagnosis can be seen as a classification and decision-making problem based on multi-source information. Dempster-Shafer Theory (DST) [1] in information fusion has been widely applied in the fault diagnosis [2,3], because it has advantage in the representing, measurement and combination of uncertainty.

However, the existing DST-based fault diagnosis method is mainly used to deal with single fault diagnosis problem, for the fundamental assumption of "mutually exclusive" makes simultaneous faults can't be characterized in DST. But simultaneous faults (e.g. rotor unbalance and misalignment faults occur simultaneously) often occur in the practice, especially in large and complex equipments. In order to enhance the safety, stability, reliability, and reduce the cost of maintenance, we should make a deep study of simultaneous faults, so as to improve the technique of fault diagnosis continuously. In 2002, Dezert and Smarandache proposed a new theory of plausible and paradoxical reasoning, known as Dezert-Smarandache Theory (DSmT) [4], which is the extension of DST. Compared with the DST, DSmT can not only represent and deal with the uncertainty easily, but also can describe and handle the conflict properly. The frame of discernment of DSmT relaxes the exclusivity constraint in DST, it means the elements in this frame do not need to satisfy "mutually exclusive". Therefore, simultaneous faults can be represented by the intersection of elements in DSmT.

On the other hand, both DSmT and DST require that evidence must be independent [5], which is a strict limitation that can not be satisfied in practical application. In an intelligent multi-sensor system, the sensors are different in the time, space, credibility, but they closely contact with various features of environment, equipment, information processing method and other factors, therefore, the information acquired by sensors will be dependent with each other. But it usually is regarded as independent evidence approximately [6-8] which often leads to the over-estimation of result, such as the normal state will be judged as failure.

This paper provides a method based on DSmT for simultaneous faults diagnosis when evidence is dependent. Firstly, according to the characteristics of simultaneous faults, a frame of discernment is given for both single fault and simultaneous faults diagnosis, the DSmT combination rule applicable to simultaneous faults diagnosis is introduced. Secondly, the dependence of original evidence is classified according to three main factors: sensors, types of fault features, methods for extracting of fault feature, a method for evidence decorrelation is provided. On the other hand, the weights for measuring evidence credibility are given to modify independent evidence based on Generalized Ambiguity Measure. Next, DSmT combination rule is used to aggregate the modified evidence. Finally, an example of rotor faults diagnosis is given to illustrate effectiveness of the proposed method.

2. DSmT combination model of simultaneous faults diagnosis

In DST, the frame of discernment Θ is composed of n exhaustive and exclusive element θ_i (i =1 ,2 ,…, n), defined as follows: $\Theta = \{ \theta_1 , \theta_2 ,\ldots, \theta_n \}$. The exclusiveness means $\theta_k \cap \theta_l = \varnothing$ (k , l =1,2,…, n ; $k \neq l$). In the DST-based fault diagnosis method, each element θ_i represents a fault mode. For the fundamental assumption of "mutually exclusive" makes simultaneous faults can't be characterized in DST, the DST-based fault diagnosis method only can be used in single fault diagnosis. Therefore, in order to solve the problems of simultaneous faults diagnosis, a frame of discernment for both single fault and simultaneous faults diagnosis should be build at first.

Definition 1. Generalized frame of discernment [4]

A set $\Theta = \{ \theta_1 , \theta_2 ,\ldots, \theta_n \}$ is called a generalized frame of discernment if it is a finite set of n exhaustive elements.

Obviously, the frame of discernment of DSmT relaxes the exclusivity constraint in DST, it means elements θ_i do not need to satisfy $\theta_k \cap \theta_l = \varnothing$ ($k \neq l ; k , l$ =1,2,…, n). If θ_1 and θ_2 are two fault modes, then "$\theta_1 \cap \theta_2$" means that θ_1 and θ_2 occur simultaneously. Obviously, it is reasonable to use "$\cap$" to describe two faults that occur simultaneously.

Definition 2. Hyper-power set D^Θ [4]

The hyper-power set defined in DSmT is similar to the power set 2^Θ defined in DST. Let $\Theta = \{ \theta_1 , \theta_2 ,\ldots, \theta_n \}$ be a generalized frame of discernment, the hyper-power set D^Θ is defined as the set of all composite propositions built from elements of Θ with $\cup$ and $\cap$ operators such that

(1) $\varnothing , \theta_1 , \theta_2 ,\ldots, \theta_n \in D^\Theta$;

(2) If A , $B \in D^\Theta$, then $A \cap B \in D^\Theta$ and $A \cup B \in D^\Theta$;

(3) No other elements belong to D^Θ, except those obtained by using rules (1) or (2).

The generation of hyper-power D^Θ when n =2, $\Theta = \{\theta_1, \theta_2\}$, one has D^Θ ={ $\alpha_0, \alpha_1, \cdots, \alpha_4$ } and $|D^\Theta| = 5$ with $a_0 \triangleq \varnothing$, $a_1 \triangleq \theta_1$, $a_2 \triangleq \theta_2$, $a_3 \triangleq \theta_1 \cap \theta_2$, $a_4 \triangleq \theta_1 \cup \theta_2$.

In the actual fault diagnosis, these elements all represent some certain fault modes, such as a_0 represents "fault-free", a_1 means "θ_1 occurs", a_3 expresses "θ_1 and θ_2 are concurrent", a_4 is the universal set in evidence theory, signifies "total ignorance". We can see from the above, all the faults that may exist in equipment can be described by hyper-power set reasonably. Therefore, a certain subset of hyper-power set can be seen as the proposition space of simultaneous faults.

DSmT is the extension of DST, so the generalized basic belief assignment (GBPA), generalized belief and generalized plausibility functions are defined in almost the same manner as within the DST.

Definition 3.The hybrid DSmT rule of combination [4]

The hybrid DSmT rule of combination for $k \geq 2$ independent sources of information is defined for all $A \in D^\Theta$ as

$$m_{\mathcal{M}(\Theta)}(A) \triangleq \phi(A)\left[S_1(A) + S_2(A) + S_3(A)\right] \quad (1)$$

where all sets involved in formulas are in the canonical form and ϕ (A) is the *characteristic non-emptiness function* of a set A, i.e. ϕ (A) =1 if $A \notin \varnothing$; and ϕ (A) =0 otherwise, where $\varnothing \triangleq \{\varnothing_{\mathcal{M}} , \varnothing \}$; $\varnothing_{\mathcal{M}}$ is the set of all elements of D^Θ which have been forced to be empty through the constraints of the model M and $\varnothing$ is the classical/universal empty set.

$S_1(A)$ corresponds to the free DSmT rule [4] for k independent sources and is given by

$$S_1(A) \triangleq \sum_{\substack{X_1, X_2, \cdots, X_k \in D^\Theta \\ X_1 \cap X_2 \cap \cdots \cap X_k = A}} \prod_{i=1}^{k} m_i(X_i) \quad (2)$$

$S_2(A)$ represents the mass of all relatively and absolutely empty sets which is transferred to the total or relative ignorance associated with non existential constraints [4].

$$S_2(A) \triangleq \sum_{\substack{X_1,X_2,\cdots,X_k\in\varnothing \\ [u=A]\vee[(u\in\varnothing)\wedge(A=I_t)]}} \prod_{i=1}^{k} m_i(X_i) \tag{3}$$

$S_3(A)$ transfers the sum of relatively empty sets directly onto the canonical disjunctive form of non-empty sets

$$S_3(A) \triangleq \sum_{\substack{X_1,X_2,\cdots,X_k\in D^\Theta \\ X_1\cup X_2\cup\cdots\cup X_k=A \\ X_1\cap X_2\cap\cdots\cap X_k\in\varnothing}} \prod_{i=1}^{k} m_i(X_i) \tag{4}$$

with $u \triangleq u(X_1)\cup u(X_2)\cup\cdots u(X_k)$ where $u(X)$ is the union of all θ_i that compose X, $I_t=\theta_1\cup\theta_2\cup\cdots\cup\theta_n$ is the total ignorance. $S_2(A)$ represents the mass of all relatively and absolutely empty sets which is transferred to the total or relative ignorance associated with non existential constraints; $S_3(A)$ transfers the sum of relatively empty sets directly onto the canonical disjunctive form of non-empty sets. In practical application, if some faults do not appear, they can be belonged in empty set $\varnothing$, $S_2(A)$ and $S_3(A)$ realize the redistributions of information so as to avoid the loss of useful information when these faults be set to $\varnothing$.

3. Decorrelation method for dependent sources of evidence

3.1. The degree of dependence

The degree of dependence of evidence is denoted as w_{dep} defined by two ways [6]. One is to set the degrees of dependence according to prior knowledge, such as w_{dep}=0.5 for high dependent evidence and w_{dep}=0.25 for weak dependent evidence. The other is to calculate the degrees according to the relevant factors that affect evidence obtaining. These two ways can be integrated as a new method to allocate the degree of dependence. In the diagnosis process, sensors are used to collect the fault information, there are three main factors in information acquisition and extraction: sensors, types of fault features, methods for extracting of fault feature (namely the solution for GBPA). Based on these factors, first we separate evidence into eight categories along the three factors (see Figure 1), then divide the eight categories into three classifications according to prior knowledge, their degrees of dependence are given as: high dependence (2/3), weak dependence (1/3) and independence. For example, if two pieces of evidence that obtained by using same method to the different types of fault features of the same sensor, then the degree of dependence of these two pieces of evidence is w_{dep}=2/3. For there are three degrees of freedom (method, sensor, and fault feature) and evidence that attained differs only in the feature.

According to the separating procedure, it is known that if evidence obtained by the same sensors, then there is more or less dependence between evidence; evidence is independent if it is attained by different sensors.

3.2. The decorrelation method based on degree of dependence

First, evidence is partitioned into different groups according to degrees of dependence, every evidence in each group then can be separated into overlapping sources of evidence and independent sources of evidence according to formula (5) and (6).

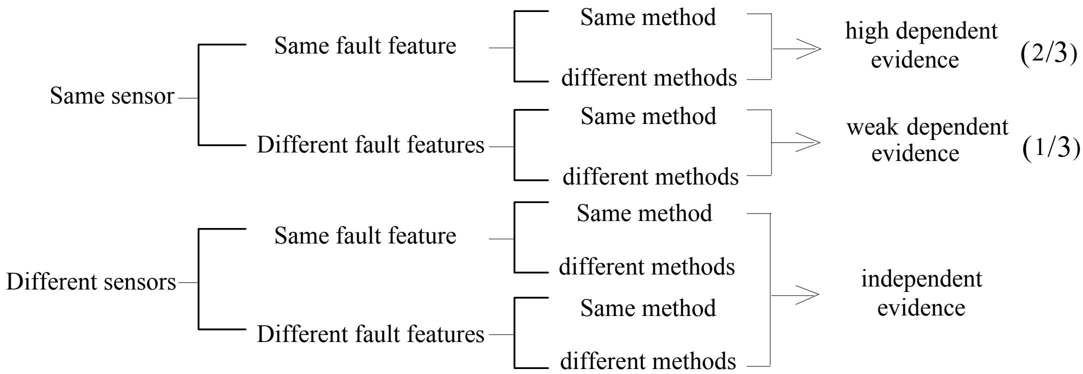

Figure 1. The separation of dependence for evidence

$$\begin{cases} m_{i,ident}(A) = w_{dep} m_{i,orig}(A) & \forall A \neq \Theta \\ m_{i,ident}(\Theta) = 1 - \sum_{\forall A \neq \Theta} m_{i,ident}(A) \end{cases} \quad (5)$$

$$\begin{cases} m_{i,ind}(A) = (1 - w_{dep}) m_{i,orig}(A) & \forall A \neq \Theta \\ m_{i,ind}(\Theta) = 1 - \sum_{\forall A \neq \Theta} m_{i,ind}(A) \end{cases} \quad (6)$$

Where $m_{i,iden}(A)$ and $m_{i,ind}$ (A) denote respectively the belief based on overlapping evidence and the belief based on independent evidence for the i th evidence in a certain group.

Suppose there are k pieces of dependent evidence in a group, after performing the separating procedure for every evidence in the group, two belief sets of evidence are gotten: one is composed of k beliefs $m_{i,iden}(A)$ (i=1, 2,…, k) based on independent evidence, the other is composed of k beliefs $m_{i,ind}(A)$ (i=1, 2,…, k) based on identical evidence. Then, the average belief of $m_{i,iden}(A)$ (i=1, 2,…, k) is

$$\overline{m}(A) = \frac{1}{k} \sum_{i=1}^{k} m_{i,ident}(A) \quad \forall A \in D^{\Theta} \quad (7)$$

Where the quantity m is an average belief assigned to A by k pieces of identical evidence. Therefore, k+1 pieces of independent evidence are attained, as in Figure 2.

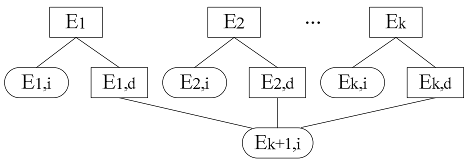

Figure 2. $E_{k,i}$-- independent sources of evidence
$E_{k,d}$--dependent sources of evidence
$E_{k+1,i}$ --average of k dependent sources of evidence

After the reconciliation step, we have a set of k+1 pieces of independent evidence.

4. The weight for measuring credibility of evidence

Reference [9] gives a Ambiguity Measure (AM) based on the Classical Pignistic transformation (CPT), which can perfectly quantify the total uncertainty of evidence.

To take a rational decision within the DSmT framework, it is necessary to generalize the CPT in order to construct a pignistic probability function by generalized basic belief assignment drawn from the DSmT combination rule. Reference [4] proposes the simplest and direct extension of the CPT to define a Generalized Pignistic Transformation (GPT) as follows

$$P(A) = \sum_{X \in D^{\Theta}} \frac{C_M(X \cap A)}{C_M(X)} m(X) \quad \forall A \in D^{\Theta} \quad (8)$$

where $C_M(X)$ denotes the DSm cardinal of proposition X for the DSm model of the problem under consideration.

Then we can define a Generalized Ambiguity Measure (GAM) based on GPT, by which, the weigth for measuring credibility of evidence can be obtained. Let Θ be a frame of discernment, m be a GBPA defined on Θ , then define the total uncertainty of evidence as

$$GAM(m) = -\sum_{A \in \Theta} P(A) \times \log_2(P(A)) \quad (9)$$

Suppose there are N pieces of independent evidence m_i (A) (i=1, 2,…,N), weights $w_i (i=1,2,\cdots,N)$ can be proposed to measure the credibility of evidence as

$$w_i = 1 - \frac{AM(m_i)}{\sum_{j=1}^{N} AM(m_j)} \quad (10)$$

According to the analysis of upper formula, we know that the larger the uncertainty GAM is, the smaller information quantity is, that is to say the smaller the weight w_i is. From section 3, there are k+1 pieces of decorrelated independent evidence, N-(k+1) pieces of original independent evidence. For the N pieces of evidence, the modified $m_i'(A)$ (i=1, 2,…, N) are

$$\begin{cases} m_i'(A) = w_i m_i(A) & \forall A \neq \Theta \\ m_i'(\Theta) = 1 - \sum_{\forall A \neq \Theta} m_i(A) \end{cases} \quad (11)$$

5. Simultaneous faults diagnosis

5.1. Experiment set-up

An example from ZHS-2 multi-function rotor test-bed is given to illustrate the method of decorrelation and simultaneous faults diagnosis described in this paper is effective. We set three fault modes $\Theta = \{\theta_1, \theta_2, \theta_3\}$ (the elements in Θ are fault modes): rotor unbalance (θ_1), rotor misalignment (θ_2), rotor clamping support loosening (θ_3). There are two sensors: vibration acceleration sensor and vibration displacement sensor. These two sensors are installed on the rotor clamping support in vertical and horizontal direction respectively to collect vibration signal. Then the signal is transferred to a host computer by a data acquisition device (HG8902) and analyzed by a software installed in the host computer, then 4 types of fault features are extracted by the software: amplitude of 1~3 frequency doubling (1*X*~3*X*) of the vibration acceleration and average amplitude of the vibration displacement.

According to expert experience, rotor unbalance (θ_1) and misaligned (θ_2) faults often occur simultaneously in the test-bed, so we have $\theta_1 \cap \theta_2 \neq \varnothing$, and other simultaneous faults do not happen, so $\theta_k \cap \theta_l = \varnothing$ ($k \neq l$; k, $l =$ 1,2,3,4; and the set $\{k, l\} \neq \{1,2\}$).

Because of the operating characteristics of sensors and impact of external environment, fault feature information that obtained by sensors is usually fuzzy. Therefore, fuzzy membership function is used to describe fault features. The GBPAs of 4 pieces of evidence can be generated based on fuzzy membership function using the statistical method [10].

5.2. Decision rules

In this paper, we select the basic probability assignment function as decision function, the decision rules are given as

(1) The identified mode corresponding to the maximal GBPA value, which must be greater than a certain threshold T_1. Generally, $T_1 > 1/n$ (n is the number of the modes);

(2) The difference of the GBPAs between the identified mode and the other GBPAs must be greater than a given threshold T_2;

(3) The uncertainty $m(\Theta)$ must be less than a certain threshold T_3.

Let T_1=0.4, T_2=0.2, T_3=0.2 in this paper.

5.3. Diagnosis results

As stated earlier, amplitudes of 1~3 frequency doubling (1*X*~3*X*) are acquired from the vibration acceleration sensor, so these 3 pieces of evidence (their GBPAs are denoted as m_1, m_2, m_3 respectively) are dependent and should be handled by decorrelation process. The average amplitude of displacement (its GBPA is denoted as m_4) is gotten from vibration displacement sensor, therefore, it is independent with the above 3 pieces of evidence. According to Figure 1, the degree of dependence w_{dep}=1/3. The independent evidence after decorrelation and original independent evidence from

Table 1. Diagnosis results of DSmT combination rule

GBPA	Evidence				DSmT
	m_1	m_2	m_3	m_4	
$m(\theta_1)$	0.3865	0.4108	0.2364	0.4476	0.0811
$m(\theta_2)$	0.4043	0.4390	0.3405	0.4473	0.1078
$m(\theta_3)$	0.2068	0.0697	0.3449	0.0021	0.0195
$m(\theta_1 \cap \theta_2)$	—	—	—	—	0.6710
$m(\theta_1 \cup \theta_3)$	0.0000	0.0000	0.0000	0.0000	0.0304
$m(\theta_2 \cup \theta_3)$	0.0000	0.0000	0.0000	0.0000	0.0341
$m(\theta_3 \cup (\theta_1 \cup \theta_2))$	—	—	—	—	0.0561
$m(\Theta)$	0.0024	0.0805	0.0782	0.1031	0.0000
Results	uncertain	uncertain	uncertain	uncertain	$\theta_1 \cap \theta_2$

Table 2. Diagnosis results of proposed method

GBPA	Evidence					Combination result
	m'_1	m'_2	m'_3	m'_4	m'_5	
$m(\theta_1)$	0.2016	0.2207	0.1209	0.0917	0.3787	0.1785
$m(\theta_2)$	0.2109	0.2357	0.1742	0.1050	0.3785	0.2073
$m(\theta_3)$	0.1078	0.0375	0.1764	0.0551	0.0018	0.0247
$m(\theta_1 \cap \theta_2)$	—	—	—	—	—	0.4474
$m(\theta_1 \cup \theta_3)$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0558
$m(\theta_2 \cup \theta_3)$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0581
$m(\theta_3 \cup (\theta_1 \cup \theta_2))$	—	—	—	—	—	0.0052
$m(\Theta)$	0.4797	0.5061	0.5284	0.7482	0.2410	0.0231
Results	uncertain	uncertain	uncertain	uncertain	uncertain	$\theta_1 \cap \theta_2$

displacement are modified by weight w_i according to formula (11), w_1 =0.7823, w_2 =0.8057, w_3 =0.7674, w_4 =0.7984, w_5 =0.8462. So the GBPAs of 4 pieces of evidence after decorrelation and weight modification are $m'_1, m'_2, m'_3, m'_4, m'_5$ shown in Table 2.

The GBPAs of 4 pieces of evidence obtained according to reference [10] and their DSmT combination results are shown in Table 1. The DSmT combination rule can diagnose that the rotor unbalance and misaligned faults have occurred simultaneously, this result is consistent with the fault mode we have set.

For the proposed method, the combination results are shown in Table 2. From the table, it can be seen that the proposed method not only can diagnose that the rotor unbalance and misaligned faults have occurred simultaneously, but also considers the evidence dependence and credibility before combination, so it is more reasonable and effective in practical application.

In the hyper-power set, single faults can be seen as the special cases of simultaneous faults, so the proposed method also can be used to diagnose the single faults. Since lack of paper space, results of single faults diagnosis are not been given. Set the test-bed in normal states, 100 statistical experiments are made. Because of DSmT combination rule does not consider the dependence between evidence, which leads to 15% of the misjudgment rate because of over-estimation, but the misjudgment rate of the proposed method is just 5%.

6. Conclusions

There are two problems in application of fault diagnosis method based on evidence theory. One is that the traditional DST-based fault diagnosis method only can be used to deal with single fault diagnosis problem, which is useless to diagnose simultaneous faults. The other is that evidence acquired by sensors usually be dependent with each other. In this paper, a combination method for dependent evidence is provided and its application in rotor for simultaneous faults diagnosis is given. Firstly, we analyzed the dependency of evidence and a method for evidence decorrelation was provided; on the other hand, the weights for measuring evidence credibility were given to modify the independent evidence; next, the modified evidence were aggregated using DSmT combination rule; finally, the example of rotor faults diagnosis showed the effectiveness and reliability of the proposed method.

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